

# Effective growth of matter density fluctuations in the running $\Lambda$ CDM and $\Lambda$ XCDM models

**Javier Grande**<sup>1</sup>, **Reuven Opher**<sup>2</sup>, **Ana Pelinson**<sup>1</sup>, **Joan Solà**<sup>1,3</sup>

<sup>1</sup> *High Energy Physics Group, Dept. ECM, Universitat de Barcelona,  
Diagonal 647, 08028 Barcelona, Catalonia, Spain*

<sup>2</sup> *IAG, Universidade de São Paulo, Rua do Matão 1226,  
Cidade Universitária, CEP 05508-900, São Paulo, SP, Brazil*

<sup>3</sup> *Institut de Ciències del Cosmos, UB, Barcelona.*

*E-mails: jgrande@ecm.ub.es, opher@astro.iag.usp.br, apelinson@ecm.ub.es,  
sola@ifae.es*

**ABSTRACT:** We investigate the matter density fluctuations  $\delta\rho_M/\rho_M$  for two dark energy (DE) models in the literature in which the cosmological term  $\Lambda$  is a running parameter. In the first model, the running  $\Lambda$ CDM model, matter and DE exchange energy, whereas in the second model, the  $\Lambda$ XCDM model, the total DE and matter components are conserved separately. The  $\Lambda$ XCDM model was proposed as an interesting solution to the cosmic coincidence problem. It includes an extra dynamical component, the “cosmon”  $X$ , which interacts with the running  $\Lambda$ , but not with matter. In our analysis we make use of the current value of the linear bias parameter,  $b^2(0) = P_{GG}/P_{MM}$ , where  $P_{MM} \propto (\delta\rho_M/\rho_M)^2$  is the present matter power spectrum and  $P_{GG}$  is the galaxy fluctuation power spectrum. The former can be computed within a given model, and the latter is found from the observed LSS data (at small  $z$ ) obtained by the 2dF galaxy redshift survey. It is found that  $b_\Lambda^2(z \simeq 0) = 1$  within a 10% accuracy for the standard  $\Lambda$ CDM model. Adopting this limit for any DE model and using a method based on the effective equation of state for the DE, we can set a limit on the growth of matter density perturbations for the running  $\Lambda$ CDM model, the solution of which is known. This provides a good test of the procedure, which we then apply to the  $\Lambda$ XCDM model in order to determine the physical region of parameter space, compatible with the LSS data. In this region, the  $\Lambda$ XCDM model is consistent with known observations and provides at the same time a viable solution to the cosmic coincidence problem.

# 1. Introduction

For the last two decades or so, the 90-years-old history of the cosmological constant problem [1] has turned into the history of the dark energy (DE) problem [2]. A massive attempt has been underway in recent times to supersede the cosmological term  $\Lambda$  in Einstein's equations by a variety of different entities which come under the mysterious name of DE, i.e., a purported new substance or, for that matter, an effective cause that is responsible for the observed accelerated expansion of the Universe [3]. In particular, there is the popular idea that the DE is associated with a cosmological dynamical scalar field (quintessence and the like) [4, 5], which fully supplants the preeminent role played by  $\Lambda$  for a long time in different chapters of modern cosmology [1, 2]. Within this broader context, the cosmological constant (CC) has been relegated to a back seat in the cosmological scenario or, at least, has been degraded to represent just one among a host of possibilities, proposed to explain the nature of the DE. In spite of this situation, the  $\Lambda$  term and the standard model ( $\Lambda$ CDM model) of modern cosmology have been thriving rather well and have survived, essentially unscathed, the entire set of observational tests to which they have been exposed up to the present time (see [6] for a summary of the experimental situation).

This state of affairs somehow suggests that, rather than trying to completely get rid of the CC term and replace it by some Ersatz entity, perhaps it would be a better idea to keep it and try to explain some of the unsatisfactory features of the cosmological standard model in terms of possible, unsuspected dynamical features of  $\Lambda$  and/or by introducing other dynamical complements to it. For example, while it is very hard to accept a small and strictly constant value of  $\Lambda$  throughout the entire history of the Universe, a slowly evolving DE looks more promising. Actually, this potentially dynamical character of the DE is the main motivation for introducing quintessence-like ideas [5]. However, the contribution from the vacuum energy, most likely represented by the  $\Lambda$  term, is still there and remains a good candidate to be considered. Therefore, instead of exchanging it for a dynamical new object, it seems more economical to just admit that  $\Lambda$  may hide some small evolution (“running”) with time or energy. Even more ambitiously, one may entertain the possibility that the CC could be a running parameter in quantum field theory (QFT) in curved space-time, i.e., an effective charge in the sense of the renormalization group (RG). We may call this scenario the “running  $\Lambda$ CDM model”. An interesting proposal along these lines was developed in [7, 8, 9] (for a review, see e.g., [10]), and was put to the test in [11, 12, 13] (see also [14, 15, 16]).

A particularly acute cosmological problem, that could be alleviated by introducing

a dynamical DE, is known as the “coincidence problem” [17], i.e., why do the matter and DE densities happen to be of the same order of magnitude, precisely in the current Universe. For example, why is the latter not one thousand or one million times smaller at present? The popular idea of a dynamical scalar field replacing the cosmological constant  $\Lambda$  was largely motivated by the possibility of having a framework where one could try to solve this conundrum. Another option for tackling this problem is to start with the running  $\Lambda$ CDM model, which does not deviate much from the standard cosmological model, and add to it a dynamical entity,  $X = X(t)$ , which was called the “cosmon” [18]<sup>1</sup>. In contrast to the standard quintessence point of view (where  $\Lambda$ CDM is replaced by  $X$ CDM), here the  $\Lambda$  term is not substituted by a scalar field. Instead, it is assumed that there may be multiple sources of DE (perhaps some fundamental and others effective), including the vacuum energy, which is tied to  $\Lambda$ . The other may be collectively represented by the effective entity  $X$ . In this model, which was called the  $\Lambda$ XCDM model in [18], matter and the total DE are conserved separately. However, the DE density here is not just  $\rho_\Lambda = \Lambda/(8\pi G)$ , but is the sum of  $\rho_\Lambda$  and the cosmon density,  $\rho_X$ , i.e.,  $\rho_D = \rho_\Lambda + \rho_X$ . This total DE is locally and covariantly conserved with the expansion of the Universe. The advantage of upgrading the  $\Lambda$ CDM model into the  $\Lambda$ XCDM is that it allows for the possibility of dynamical interplay between  $X$  and  $\Lambda$  within a scenario where the total matter and DE densities are individually conserved<sup>2</sup>. This interplay is essential in order to provide a solution for the coincidence problem as well as to allow for the  $\Lambda$ XCDM model to mimic the standard  $\Lambda$ CDM model at the present time. Indeed, for a wide range of cosmological redshifts (including the full span accessible to supernovae data), the effective DE pressure and density ( $p_D, \rho_D$ ) in the  $\Lambda$ XCDM model may simulate a constant  $\Lambda$  behavior,  $p_D \simeq -\rho_D$ , to an arbitrary high degree of approximation [18].

It should be emphasized that, in contrast to the quintessence point of view, the entity  $X$  in the  $\Lambda$ XCDM model need not be a scalar field. In fact, no physical substratum (e.g. a physical fluid) is assumed behind it. The essential condition defining  $X$  is the local covariant conservation of the total DE,  $\rho_D = \rho_\Lambda + \rho_X$ , independent of matter. In particular,  $X$  could be the effective behavior of a more complete theory, comprising Einstein’s gravity as a particular case. For example, it could represent the

---

<sup>1</sup>See [19, 20] for alternative cosmological models involving the cosmon and both variable cosmological term  $\Lambda$  and Newton’s coupling  $G$ .

<sup>2</sup>The name *cosmon* was first introduced in [21]. Here we use it in a generalized sense for any additional component(s) of the DE, other than  $\Lambda$ , provided that the total DE density remains covariantly conserved.

net behavior of higher order terms in the effective action.

An essential aspect to address in any proposed model for the DE is its ability to reproduce the existing observations. The  $\Lambda$ XCDM model has been already put to the test concerning nucleosynthesis bounds and the equation of state (EOS) behavior at small and large redshifts, that are relevant to supernovae and CMB data [18]. However, one of the most important tests yet to be made is the verification of the existence of a region of parameter space that is also compatible with the data on the large scale structure (LSS) formation. This is, in fact, the main aim of the present paper. The LSS is contained in the galaxy fluctuation power spectrum  $P_{GG}$  which has been measured by the 2 degree Field Galaxy Redshift Survey (2dFGRS), for example, [22]. It is remarkable that the joint 2dFGRS and CMB analysis presented in [23] shows that there is a good agreement of the LSS data with the measurements of the CMB anisotropies for the  $\Lambda$ CDM model as well as with numerical simulations of galaxy formation [24].

On the theoretical side, this data must be reproduced by the predicted matter power spectrum,  $P_{MM}$ , for any successful model of structure formation. Therefore, for very large scales, it is to be expected that the linear bias parameter,  $b^2 \equiv P_{GG}/P_{MM}$  [23], should behave as a definite, scale-independent, quantity at small cosmological redshifts, i.e., when the distribution of galaxies had enough time to be correlated with the mass distribution in the Universe. This is, in fact, what is obtained in the successful standard  $\Lambda$ CDM model, where the observed present value of  $b^2$  turns out to be 1 within a 10% accuracy [22, 23]. It seems reasonable to extend this criteria to any other cosmological model aiming at a good description of the presently observed LSS. For example, in the power spectrum  $P_{MM}$  for the running  $\Lambda$ CDM model [7, 8], which has been fully studied in [25], the matter fluctuations have been solved in a framework where they are coupled with the perturbations in the DE, in this case represented by the running  $\Lambda$ , described by the parameter  $\nu$ . Its comparison with the galaxy fluctuation power spectrum  $P_{GG}$  [22] puts stringent limits on the fundamental parameter  $\nu$  of this model. A non-vanishing value of  $\nu$  produces a time evolution of  $\Lambda$ . The explicit solution of the model shows that values  $|\nu| > 10^{-4}$  are most likely excluded [25].

Here, we wish to address the LSS test for the  $\Lambda$ XCDM model. The test includes three parameters:  $\nu$  (the time evolution of  $\Lambda$ ),  $w_X$  (the effective barotropic index of the cosmon), and  $\Omega_\Lambda^0$  (the current CC density in units of the critical density). In this model as well, a non-vanishing value of  $\nu$  entails an evolving  $\Lambda$ , but in contradistinction to the running  $\Lambda$ CDM model or the interactive quintessence models [26], there is no crosstalk between matter and DE, which is an attractive feature. The exact analysis

of matter density perturbations is more complicated in the  $\Lambda$ XCDM model, and we tackle it using the following two-step “effective method”. First of all, we note that it is possible to ascribe an effective EOS to a variable  $\Lambda$  model. This has been explored in detail in [12, 13]. Using these results, we apply the effective EOS approach to the matter perturbation equations following [27, 28, 29]. This enables us to obtain an approximate treatment of the growth of matter perturbations, in which the DE perturbations are neglected and all the DE effects are encoded in the effective EOS,  $p_D = w_e \rho_D$  and in the ratio of DE to matter densities,  $r = \rho_D/\rho_M$ . Secondly, to obtain useful bounds on the parameters of the model, we use the linear bias parameter,  $b^2(z) = P_{GG}/P_{MM}$ . Specifically, we impose the condition (“F-test” [30, 31]) that its value cannot deviate from the  $\Lambda$ CDM value by more than 10% at  $z = 0$ . From the above, the F-test should be essentially equivalent (although not identical) to requiring that  $b^2(0) = 1 \pm 0.1$  [22, 23]. Some concrete applications of this test can be found in [30]. In the present paper we look for the viable physical region of parameter space for the  $\Lambda$ XCDM model, using the aforementioned effective method. However, to check its efficiency when applied to non-trivial models with variable  $\Lambda$ , we first address the running  $\Lambda$ CDM model. It is important to emphasize that, in the running  $\Lambda$ CDM model, the DE (represented by the variable CC) and matter fluids exchange energy and, therefore, are interacting components of the cosmic medium. Fortunately, since we know the results of a full-fledged treatment of density perturbations (of both matter and DE) in this model [25], we can compare them with those obtained from the effective approach, in which DE and matter are treated as conserved, non-interacting components and the perturbations of the DE are neglected. This effective approach is meaningful, provided we arrange that the expansion history is the same as that of the original running  $\Lambda$ CDM model. We may call this alternative representation the “DE picture” [13] because it is close to the standard quintessence representation of the DE [5]. The comparison provides an excellent test for the effective method used in the DE picture. Finally, we apply the effective EOS procedure, in combination with the F-test, to the more complicated situation of the  $\Lambda$ XCDM model and obtain the corresponding region of parameter space that is compatible with the LSS data. The existence of this region strengthens, once more, the viability and likelihood of the  $\Lambda$ XCDM model as a robust solution of the coincidence problem [18, 19, 20].

The structure of the paper is as follows. In the next section, we review the effective EOS approach to the computation of matter density perturbations and discuss the F-test. In section 3, we apply the effective EOS approach and the F-test to the running

$\Lambda$ CDM model. In sections 4 and 5 we describe the  $\Lambda$ XCDM model and present a detailed numerical analysis of the matter density perturbations in order to determine the physical region of parameter space. The final section is devoted to discussion and conclusions.

## 2. Dark energy, density fluctuations and the F-test

In this section, we discuss the effective approach to the computation of the linear matter density fluctuations for dark energy (DE) models in which the matter components are canonically conserved [27, 28, 29], as well as the definition of the F-test [30]. The main aim is to lay the groundwork for section 3, where we apply them to a cosmological model with a variable cosmological term  $\Lambda$ , for which an effective equation of state (EOS) can be defined [12, 13]. For a general DE model within the flat Universe, the Friedmann equation can be written in terms of the normalized matter and DE densities as

$$\frac{H^2(a)}{H_0^2} = \Omega_M(a) + \Omega_D(a), \quad (2.1)$$

where  $H_0$  is the present value for the Hubble parameter and  $a(z) = 1/(1+z)$  is the scale factor in terms of the cosmological redshift  $z$ . The matter and DE densities are normalized in terms of the present critical density  $\rho_c^0 \equiv 3H_0^2/8\pi G$ :

$$\Omega_M(a) \equiv \frac{\rho_M(a)}{\rho_c^0}, \quad \Omega_D(a) \equiv \frac{\rho_D(a)}{\rho_c^0}. \quad (2.2)$$

It will prove useful to also define the set of “instantaneous” cosmological parameters at cosmic time  $t$ , when the scale factor was  $a = a(t)$ ,

$$\tilde{\Omega}_M(a) = \frac{\rho_M(a)}{\rho_c(a)}, \quad \tilde{\Omega}_D(a) = \frac{\rho_D(a)}{\rho_c(a)}, \quad (2.3)$$

where  $\rho_c(a) = 3H^2(a)/8\pi G$  is the critical density at the same instant of cosmic time  $t$ . These parameters should not be confused with those in (2.2). We will use both sets, depending on the situation, and for this reason, the parameters in (2.3) are written with a tilde. Notice that only these parameters satisfy the normalized cosmic sum rule at any time:

$$\tilde{\Omega}_M(a) + \tilde{\Omega}_D(a) = 1. \quad (2.4)$$

While the parameters of the original set (2.2) also add up to one at  $t = t_0$  (present time), they satisfy the relation (2.1), in general.

If matter and DE are individually conserved (“self-conserved”), we can split the overall conservation law into two equations,

$$\rho'_M(a) + \frac{3}{a}\rho_M(a) = 0, \quad (2.5)$$

$$\rho'_D(a) + \frac{3}{a}(1 + w_e)\rho_D(a) = 0, \quad (2.6)$$

where the primes indicate differentiation with respect to the scale factor,  $f' \equiv df/da$ . In the equations above we have parameterized the relation between the pressure and density of the dark energy and matter as follows:

$$p_D = w_e \rho_D, \quad p_M = 0, \quad (2.7)$$

where  $w_e$  is the effective equation of state (EOS) “parameter” of the DE. It is not, in general, a constant parameter, but a function of the scale factor  $a$ , namely  $w_e \equiv w_e(a)$ . Therefore, we do not expect the DE to behave as a simple barotropic fluid. The solution of (2.6) can be expressed in terms of the normalized DE density defined in (2.2):

$$\Omega_D(a) = \Omega_D^0 \exp \left\{ -3 \int_1^a \frac{da'}{a'} [1 + w_e(a')] \right\}, \quad (2.8)$$

where  $\Omega_D^0 = \Omega_D(a = 1)$  is the normalized DE density at present. Therefore, the EOS coefficient is obtained from

$$w_e(a) = -1 - \frac{a}{3} \frac{1}{\Omega_D(a)} \frac{d\Omega_D(a)}{da}. \quad (2.9)$$

Notice that we consider only non-relativistic, pressureless, matter because our perturbation calculation refers only to the epoch of structure formation. Since the matter conservation law is decoupled, the normalized matter density following from (2.5) reads

$$\Omega_M(a) = \Omega_M^0 a^{-3}, \quad (2.10)$$

where  $\Omega_M^0$  is the present normalized total matter density.

Our analysis of matter density perturbations applies after the radiation dominated era. Thus, we take  $z$  (alternatively  $a$ ) from about the recombination epoch  $z_{\text{rec}} \simeq 1100$  (i.e.  $a \simeq 10^{-3}$ ) to  $z = 0$  ( $a = 1$ , today). Moreover, it applies only to sufficiently large scales, where the perturbations follow the linear regime, as we discuss below. On scales within the horizon, fluctuations in the dark energy density disperse relativistically and the DE component becomes smooth, i.e., the density perturbations  $\delta\rho_D$  can be considered negligible. In these conditions, the evolution of the matter density fluctuations

$\delta\rho_M$  with the cosmic time  $t$  can be computed from the well-known equation of motion in the Newtonian approximation [32],

$$\ddot{\delta}_M + 2H\dot{\delta}_M - 4\pi G\rho_M\delta_M = 0, \quad (2.11)$$

where, again, pressure effects (in this case pressure perturbations) are neglected in the matter dominated era. Here  $G$  is the Newton constant,  $\delta_M \equiv \delta\rho_M/\rho_M$  is the fractional matter density perturbation (density contrast) and  $\dot{\delta}_M \equiv d\delta_M/dt$ . Within this framework,  $\delta_M$  is the linear growth of density fluctuations.

The previous equation is also valid in the presence of the DE, provided that  $\delta\rho_D$  is indeed negligible, as assumed. In this case,  $H$  is, of course, affected by the corresponding smooth background density contribution from  $\rho_D$ . By the same token the above equation is also valid in the presence of the spatial curvature term,  $K/a^2$ , since it can also be treated as a smooth source (similarly to the DE). Although we will restrict ourselves to the flat space case, the following reformulation of (2.11) is valid for any spatial curvature. Let us write the general Friedmann's equation as

$$H^2(a) = \frac{8\pi G}{3} \left[ \rho_M(a) + \rho_D(a) - \frac{3K}{8\pi G a^2} \right] = \frac{8\pi G}{3} \rho_c(a) = \frac{8\pi G}{3} \frac{\rho_M(a)}{\tilde{\Omega}_M(a)}, \quad (2.12)$$

where  $\tilde{\Omega}_M(a)$  was defined in (2.3). It is then easy to see that Eq. (2.11) can be written as

$$\ddot{\delta}_M + 2H\dot{\delta}_M - \frac{3}{2}H^2\tilde{\Omega}_M(a)\delta_M = 0. \quad (2.13)$$

We wish to solve this equation for some non-trivial scenarios. First, we introduce some cosmetic changes in (2.13), which prove very useful for practical purposes [29], as they allow us to apply this method to variable  $\Lambda$  models [12, 13].

We start from (2.13), for  $\delta_M$ , which, interestingly enough, can be conveniently recast such that the effective equation of state (EOS) of the DE in the given model appears explicitly. We first trade the derivative with respect to the cosmic time for the derivative with respect to the cosmic factor:  $\dot{\delta}_M = aH\delta'_M$ , where  $\delta'_M \equiv d\delta_M/da$ . Similarly,  $\ddot{\delta}_M = (aH^2 + a\dot{H})\delta'_M + (aH)^2\delta''_M$ . We observe that we can eliminate  $\dot{H}$  (i.e. the time variation of the Hubble function) from the last equation by noting that, in the matter dominated epoch, it can conveniently be written as (henceforth, we confine ourselves to the flat case only)

$$\dot{H} = -4\pi G [\rho_M + (1 + w_e)\rho_D] = -\frac{3}{2}H^2 \left[ 1 + \frac{w_e r}{1 + r} \right], \quad (2.14)$$



where  $w_e = p_D/\rho_D$  is the aforementioned effective EOS parameter of the DE and  $r = r(a)$  is the ratio between the DE and matter densities at any given time [18]:

$$r(a) = \frac{\Omega_D(a)}{\Omega_M(a)} = \frac{\tilde{\Omega}_D(a)}{\tilde{\Omega}_M(a)} = \frac{\rho_D(a)}{\rho_M(a)}. \quad (2.15)$$

The relation (2.15) will play a relevant role in the study of the cosmic coincidence problem within the  $\Lambda$ XCDM model (see section 4). Substituting the previous equations into Eq. (2.13), we finally obtain

$$\delta_M''(a) + \frac{3}{2} \left[ 1 - \frac{w_e(a) r(a)}{1 + r(a)} \right] \frac{\delta_M'(a)}{a} - \frac{3}{2} \frac{1}{1 + r(a)} \frac{\delta_M(a)}{a^2} = 0. \quad (2.16)$$

This will be our master equation to evaluate the “effective” growth of linear density fluctuations. As we said above, we assume that the DE component becomes smooth within the horizon. The DE effects enter our calculations only through the effective EOS function (2.9) and the ratio (2.15), and, thus, the linear growth of matter fluctuations is computed in an effective way. We also assume that the DE density was negligible at the recombination era and remained so until  $z \lesssim 10$ , when all relevant modes for LSS formation had already entered the horizon. In particular, the perturbation amplitude at recombination, when the CMB was formed, is independent of the particular DE model since  $\rho_D \ll \rho_M$  at that epoch.

To better assess the meaning of Eq. (2.16), let us first consider two simple examples. In the absence of DE ( $\tilde{\Omega}_D = 0$  and, thus,  $r = 0$ ) the growing mode solution of the above equation in the matter dominated epoch ( $a \sim t^{2/3}$ ) is very simple and well-known,  $\delta_M \sim a$ . However, in the presence of DE, the growing mode solution is more complicated. Assuming a time interval not very large such that  $\tilde{\Omega}_D$  and  $\Omega_D$  remain approximately constant, we can take  $w_e = -1$  and, hence, (2.16) reads

$$\delta_M''(a) + \frac{3}{2}(1 + \tilde{\Omega}_D) \frac{\delta_M'(a)}{a} - \frac{3}{2}(1 - \tilde{\Omega}_D) \frac{\delta_M(a)}{a^2} = 0. \quad (2.17)$$

Looking for a power-law solution,  $\delta \sim a^p$ , in the limit  $\tilde{\Omega}_D \ll 1$ , we find

$$\delta_M \sim a^{1-6\tilde{\Omega}_D/5} \sim a \left( 1 - \frac{6}{5} \tilde{\Omega}_D \ln a \right). \quad (2.18)$$

This equation conveys, very clearly, the physical idea of growth suppression when a (positive) DE density is present within the horizon. Although  $\tilde{\Omega}_D$  is not constant in general, this qualitative feature should persist in more realistic situations. Furthermore, if the DE has some smooth dynamical behavior, we cannot exclude that  $\tilde{\Omega}_D$  could have been negative for some period in the past. If so, the previous equation also shows

that structure formation was reinforced during that period. In this paper, we wish to study the exact numerical solutions of (2.16) in some non-trivial scenarios, where the cosmological term is not only arbitrary and non-vanishing, but is evolving smoothly with time. Specifically, we wish to solve (2.16) for both the running  $\Lambda$ CDM and  $\Lambda$ XCDM models, mentioned in the introduction. The clue to doing this is to mimic these variable  $\Lambda$  models through a non-trivial effective EOS,  $w_e = w_e(a)$ .

From the above discussion, it follows that the linear growth should essentially behave as  $\delta_M = a$  near  $z = z_{\text{rec}}$ . We are going to use this property as an exact boundary condition for solving (2.16), together with  $\delta'_M = 1$  at recombination. Equivalently, if we introduce the standard linear growth suppression factor,  $\mathcal{G}(a) = \delta_M(a)/a$ , Eq. (2.16) is readily seen to transform into

$$\mathcal{G}''(a) + \left[ \frac{7}{2} - \frac{3}{2} \frac{w_e(a) r(a)}{1 + r(a)} \right] \frac{\mathcal{G}'(a)}{a} + \frac{3}{2} \frac{[1 - w_e(a)] r(a)}{1 + r(a)} \frac{\mathcal{G}(a)}{a^2} = 0, \quad (2.19)$$

which we solve with the boundary conditions  $\mathcal{G}(z_{\text{rec}}) = 1$  and  $\mathcal{G}'(z_{\text{rec}}) = 0$ .

To compare our predictions with observations, it is useful to invoke the linear bias factor. Its present value is defined as  $b^2(0) = P_{GG}/P_{MM}$ , where  $P_{MM}$  is the computed matter power spectrum at the present epoch, starting from the  $P_{MM}$  at the recombination epoch (obtained from the observed CMB) and  $P_{GG}$  is the galaxy distribution power spectrum (obtained from the observed galaxy-galaxy correlation function). Therefore,  $b^2(0)$  measures the difference in clustering between galaxies and mass fluctuations, i.e., it parameterizes the degree by which light traces mass. It can be related to the rms mass fluctuations on random spheres of radius  $R h^{-1}$  Mpc, typically with  $R = 8$ , and hence connected with  $\sigma_8$  [32]. In general, the bias factor can also be defined in the non-linear regime. Here, however, we are concerned only with the *linear* bias factor [23], which measures the difference in clustering between galaxies and mass fluctuations at very large scales, namely at scales for which the wave-numbers of the Fourier modes are in the range  $0.02 < k < 0.15 h \text{ Mpc}^{-1}$ . The observational data concerning the linear regime do, in fact, lie in this range [22]. Expressing the scales in terms of the Hubble radius  $H_0^{-1} \simeq 3000 h^{-1} \text{ Mpc}$  ( $h \simeq 0.7$ ), this implies values of  $k$  up to  $450 H_0$ . Thus, the minimum length scale explored by the available LSS data is of order of  $10 \text{ Mpc} \sim 8 h^{-1} \text{ Mpc}$  spheres.

For the computation of the linear bias factor, we need the matter power spectrum, whose general structure is

$$P_{MM}(k, a) \propto \left( \frac{\delta \rho_M}{\rho_M} \right)^2 = \delta_M^2(k, a) = a^2 \mathcal{G}^2(k, a), \quad (2.20)$$

where  $\delta_M(k, a)$  and  $\mathcal{G}(k, a)$  are the Fourier transforms of the corresponding quantities. In general, the correlation function for the mass distribution need not coincide with the correlation function for galaxies. Rather, a “bias” between the two is expected [32]. However, at the LSS level, it is also expected that the value of the linear bias should be some scale-independent number in the late epochs of structure formation ( $z \simeq 0$ ), namely when galaxies have had time enough to be correlated with the mass, or equivalently, when the gravitational pull has drifted them to overdense regions. As a matter of fact, the observed galaxy power spectrum  $P_{GG}$ , emerging from the final 2dFGRS catalog, did indicate these features very clearly. Most remarkably, the data pointed to the value  $b_\Lambda^2(z \simeq 0) = 1.0$  to within a 10% accuracy for the  $\Lambda$ CDM model [22], i.e., when  $P_{MM}$  in (2.20) is computed from the growth factor for the standard cosmological model, characterized by strictly constant  $\Lambda$ . This is in agreement with the previous result,  $b(L_S, z \simeq 0) = 1.10 \pm 0.08$ , of the 2dFGRS collaboration for the APM-selected massive galaxies ( $L_S = 1.9L_*$ ), averaged over all types [23], indicating that there is one  $L_*$  galaxy per dark matter halo of mass  $\sim 10^{13} h^{-1} M_\odot$  at the present epoch.

As indicated before, in the evaluation of the bias factor,  $P_{GG}$  is fixed by the LSS data, whereas  $P_{MM}$  is a theoretical quantity – and, hence, model dependent. However, the very good prediction (at the 10% level) of the bias factor by the  $\Lambda$ CDM near  $b^2(0) = 1$ , suggests the following strategy to put limits on new models of structure formation. Rather than computing  $P_{MM}$  in detail for the given model, it can just be compared to the  $\Lambda$ CDM model. This is, of course, simpler than the full computation of  $P_{MM}$ , in which details of the transfer function and other normalization factors must be included in the prefactor of (2.20). These prefactors are common and cancel in the ratio. Therefore, in the present analysis, we adhere to the following “F-test” (first proposed in [30]), which uses the method of comparison to evaluate the viability of a given DE model. To gauge the deviation of the power spectrum of the model,  $P_{MM}(a)$ , with respect to that of the  $\Lambda$ CDM model,  $P_{MM}^\Lambda(a)$ , we define the parameter:

$$|F| \equiv \left| \frac{P_{MM}(a) - P_{MM}^\Lambda(a)}{P_{MM}(a)} \right|_{a=1} = \left| \frac{\mathcal{G}^2(a) - \mathcal{G}_\Lambda^2(a)}{\mathcal{G}^2(a)} \right|_{a=1}, \quad (2.21)$$

where  $\mathcal{G}(a)$  is the solution of (2.19) for the given DE model with some effective EOS  $w_e = w_e(a)$  (in some cases  $w_e$  can be constant, but in general, it is a function of the scale factor), and  $\mathcal{G}_\Lambda(a)$  is the corresponding solution for  $w_e = -1$  ( $\Lambda$ CDM). From its definition, the factor  $F$  is a number, computed for  $a = 1$  (i.e., the current Universe at  $z = 0$ ). On the other hand, since we compare all models to the same observed galaxy

power spectrum  $P_{GG}$ , it follows that the above F-factor can be directly related to the relative difference between the linear bias factor of the given model with that of the  $\Lambda$ CDM model:

$$|F| = \left| 1 - \frac{P_{MM}^\Lambda(z)}{P_{MM}(z)} \right|_{z=0} = \left| 1 - \frac{b^2(z)}{b_\Lambda^2(z)} \right|_{z=0} . \quad (2.22)$$

As emphasized above, observations [22, 23] show a scale invariant linear evolution of  $b_\Lambda^2(z)$  for the  $\Lambda$ CDM model towards  $b_\Lambda^2(0) = 1.0 \pm 0.1$  at present. Therefore, since all the DE models should, presumably, approach the predictions of the successful  $\Lambda$ CDM model near the present time, we require that any given DE model with a growth factor  $\mathcal{G}(a)$  should pass the following ‘‘F-test’’ [30]:

$$|F| = \left| 1 - \frac{\mathcal{G}_\Lambda^2(a)}{\mathcal{G}^2(a)} \right|_{a=1} \leq 0.1 . \quad (2.23)$$

In performing the test, both  $\mathcal{G}(a)$  and  $\mathcal{G}_\Lambda(a)$  must be evolved from the recombination epoch, where the initial conditions are fixed (see above), to the present time. It is understood that the maximum limit,  $F_{\max} = 0.1$ , is saturated when  $\mathcal{G}_\Lambda^2(1)/\mathcal{G}^2(1) = 1.1$  and that the minimum limit,  $F_{\min} = -0.1$ , when  $\mathcal{G}_\Lambda^2(1)/\mathcal{G}^2(1) = 0.9$ . In the next sections, we apply these limits to the running  $\Lambda$ CDM and  $\Lambda$ XCDM models in order to bound their respective parameter spaces.

### 3. Effective method approach to the running $\Lambda$ CDM model

The F-test, defined in the previous section, is *not* exactly equivalent to requiring that  $b^2(0) = 1 \pm 0.1$  for a given model, but it is not very different from it and has the advantage of being a relatively economical procedure. However, we need to check its efficiency in some non-trivial situation before applying it to a complex DE model, such as the  $\Lambda$ XCDM model. To this end, we first apply the effective approach to the computation of matter fluctuations, together with the F-test, to the running  $\Lambda$ CDM model, for which we know the results of a complete analysis [25].

In the running  $\Lambda$ CDM model, the CC, or equivalently, its associated energy density  $\rho_\Lambda = \Lambda/8\pi G$ , is an evolving parameter. It ‘‘runs’’ because of the quantum loop effects of the high energy fields and, therefore, it satisfies a renormalization group (RG) equation. We refer the reader to the original literature for details [7, 8, 11]. Here, we just highlight the basic concepts and equations, needed for our analysis. In this framework, the physical RG running energy scale is identified with  $H$  (the Hubble parameter) and the solution of the aforementioned RG equation reads

$$\rho_\Lambda(H, \nu) = \rho_\Lambda^0 + \frac{3\nu}{8\pi} M_P^2 (H^2(a; \nu) - H_0^2) , \quad (3.1)$$

i.e., the DE density (in this case a running CC term) is an affine quadratic law in  $H$ , where  $\rho_\Lambda^0$  and  $H_0$  are the current values of these parameters. In this model we have a single (dimensionless) new parameter  $\nu$ , given by

$$\nu \equiv \frac{\sigma}{12\pi} \frac{M^2}{M_P^2}, \quad (3.2)$$

which is essentially the ratio (squared) of the effective mass  $M$  of the high energy fields to the Planck mass ( $M_P$ ), with fermions contributing  $\sigma = -1$  and bosons,  $\sigma = +1$ . If the effective mass  $M$  of the heavy particles, associated to some Grand Unified Theory, is just the Planck mass  $M_P$ , the parameter  $\nu$  takes the value (positive or negative, depending on  $\sigma$ )

$$\nu_0 = \frac{1}{12\pi} \simeq 2.6 \times 10^{-2}, \quad (3.3)$$

sometimes referred to as the “canonical” value. In practice, the preferred values of  $\nu$  are smaller than  $\nu_0$ , as we will see, which is, in fact, the natural situation because it corresponds to having the heavy particles at some scale nearby, but below, the Planck scale. We point out that the Hubble function of the running model is also  $\nu$ -dependent,  $H = H(a; \nu)$ . In the flat case,

$$H^2(a; \nu) = H_0^2 \left[ 1 + \Omega_M^0 \frac{a^{-3(1-\nu)} - 1}{1 - \nu} \right]. \quad (3.4)$$

For  $\nu = 0$ , we recover the standard form corresponding to a strictly constant  $\Lambda$ .

In the framework of the running  $\Lambda$ CDM model, there is energy exchange between the vacuum and matter sectors and we have the following mixed conservation law:

$$\rho'_\Lambda(a) + \rho'_M(a) + \frac{3}{a} \rho_M = 0. \quad (3.5)$$

From (3.1) and (3.5) we see that a non-vanishing value of  $\nu$  causes, not only a running of the CC density as a function of the scale factor (or the redshift),  $\rho_\Lambda = \rho_\Lambda(a; \nu)$ , but also an exchange of energy between  $\rho_\Lambda$  and  $\rho_M$ . For  $\nu = 0$ , however,  $\rho_\Lambda$  becomes constant and (3.5) boils down to the old matter conservation law (2.5). These features are also apparent from (3.1) and (3.4).

For all the new dynamical features that a variable CC term may entail, it should be clear that its EOS parameter still remains  $w_\Lambda = -1$ . In this context, we may speak of the model as being described within the “CC picture” [13], that is to say, the original formulation, in which the  $\Lambda$  term is explicit and the matter density is non-conserved. However, it may be advantageous to perform a “change of picture”, i.e., a description of the running  $\Lambda$ CDM model within the “DE picture” [13]. In the latter, we envision

the given running CC model as if it were a DE model with the same expansion history (that is, with the same numerical values of  $H$ ) but with self-conserved matter and DE densities (2.5) and (2.6). The numerical matching of the Hubble functions is essential in order to guarantee that the physical results are the same in both pictures. The basic reason for moving into the DE picture is because we wish to find a representation of our cosmological system where we can compute the matter perturbation equations through Eq. (2.16), in which the DE effects are confined only to the effective EOS  $w_e = w_e(a)$  and the ratio  $r(a) = \rho_D(a)/\rho_M(a)$ . Since, however, (2.16) stems from the original Newtonian form (2.11), which is derived under the hypothesis of total matter conservation [32], we must use a formulation of our cosmological scenario in which this condition is also satisfied. This representation is provided by the DE picture.

While this alternate formulation is perfectly possible, the fact that the matter density is non-conserved in the original CC picture, suggests that the mapping of the latter into the DE picture can only be carried out at the expense of finding a non-trivial effective EOS parameter (actually some complicated function  $w_e = w_e(a)$ ) relating the self-conserved  $\rho_D$  density and pressure  $p_D$  in the DE picture,  $p_D = w_e \rho_D$ . This non-trivial EOS function for the running  $\Lambda$ CDM model was determined in [12, 13]. For the present purposes, it is convenient to first determine the normalized DE density  $\Omega_D = \Omega_D(z; \nu)$  explicitly and then apply (2.9). Following the procedure indicated above and adjusting the two pictures such that the current values of  $\Omega_D$  and  $\Omega_\Lambda$  coincide, we find<sup>3</sup>

$$\Omega_D(a; \nu) = \Omega_\Lambda^0 + \frac{\Omega_M^0}{1 - \nu} a^{-3} (a^{3\nu} - 1 + \nu) - \frac{\nu \Omega_M^0}{1 - \nu}, \quad (3.6)$$

where  $\Omega_\Lambda^0 = 1 - \Omega_M^0$ . (A generalization of this method allowing for the mapping of any cosmological model with variable cosmological parameters from the original “CC picture” into the “DE picture” was developed in [13], to which we refer the reader for details on the entire procedure.) From this expression and (2.9), we find the desired result,

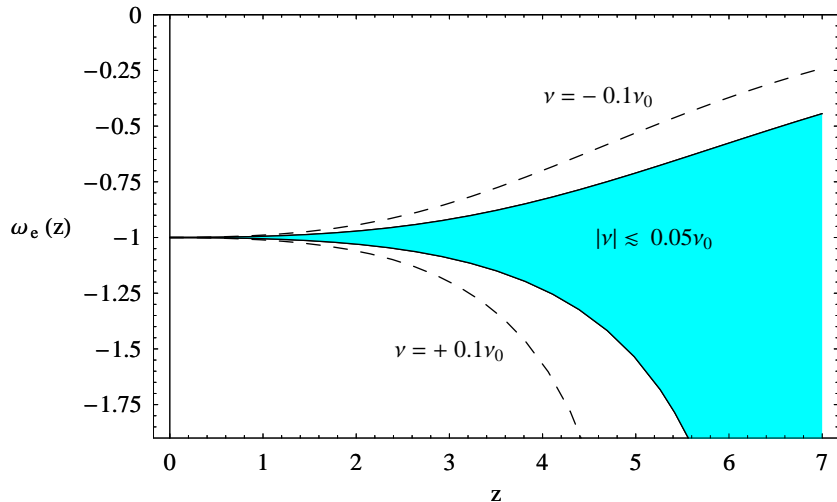
$$w_e(a; \nu) = -1 + (1 - \nu) \frac{(a^{3\nu} - 1) \Omega_M^0 a^{-3}}{1 - \nu - \Omega_M^0 + (a^{3\nu} - 1 + \nu) \Omega_M^0 a^{-3}}. \quad (3.7)$$

This effective EOS behaves nicely:  $w_e \rightarrow -1$  for  $a \rightarrow 1$ . Expanding linearly the previous formula for small  $\nu$  near our epoch, we have

$$w_e(a; \nu) \simeq -1 + 3\nu \frac{\Omega_M^0}{\Omega_\Lambda^0} \frac{\ln a}{a^3}. \quad (3.8)$$

---

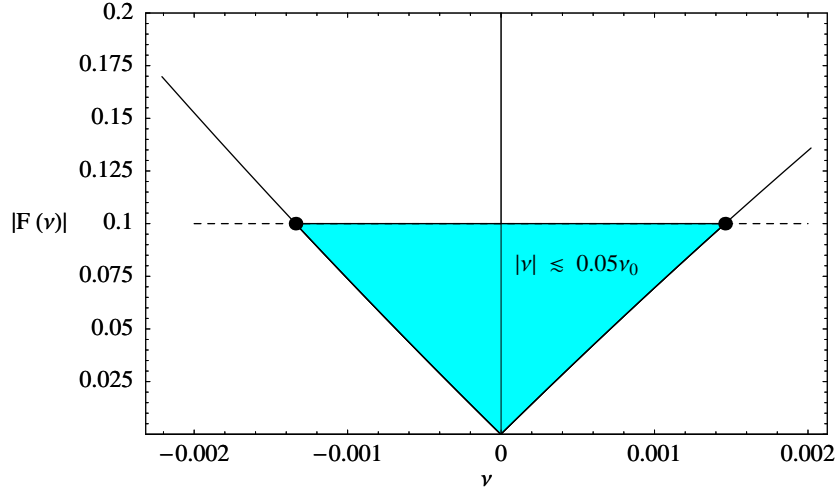
<sup>3</sup>For comparison, the corresponding normalized DE density in the original CC picture is given by (3.1), where  $H$  has the explicit form (3.4), divided by  $\rho_c^0$ . See Refs. [11] and [13] for numerical plots of these functions.



**Figure 1:** The effective EOS for the running  $\Lambda$ CDM model as a function of the cosmological redshift  $z$ . We assume flat space geometry and the prior  $\Omega_M^0 = 0.3$ . The shaded area satisfies the “F-test” condition  $|F| \leq 0.1$ , see Eq. (2.23).

Since  $a(t) < 1$  for any look-back time, this equation shows that the self-conserved density  $\rho_D$  in the DE picture for this model does not necessarily behave like quintessence ( $w_e \gtrsim -1$ ) in our past; it is only so if  $\nu < 0$ . It, however, behaves like “phantom DE” [33] (i.e.  $w_e \lesssim -1$ ) when  $\nu > 0$ .

These features can be clearly identified in Fig. 1, where we show the evolution of the effective EOS for the running  $\Lambda$ CDM model, assuming a prior  $\Omega_M^0 = 0.3$  for the normalized matter density at present. For convenience, we display  $w_e = w_e(z)$  as a function of the cosmological redshift  $z = (1 - a)/a$ . The numerical examples shown in this figure correspond to four values of  $\nu$ , expressed as small fractions of the canonical value (3.3). Specifically we plot (3.7) for  $\nu = \pm 0.1 \nu_0$  and  $\nu = \pm 0.05 \nu_0$ . Although not shown in the figure, from the analytic expression (3.7), it follows that for  $\nu > 0$ , the effective EOS for the running  $\Lambda$ CDM model goes to zero at very high redshift, whereas if  $\nu < 0$ ,  $w_e(z) \rightarrow -\nu$  for  $z \rightarrow \infty$ . We note that if  $\nu > 0$ , the denominator of Eq.(3.7) vanishes at some redshift  $z_1 > 0$ . This is related to the fact that for a positive  $\nu$ , the normalized DE density (3.6) vanishes at some point in the past. This is easily seen from the sign of this function, which changes from  $\Omega_D(a; \nu) < 0$  in the far past ( $\Omega_D \rightarrow -\Omega_M^0/a^3 < 0$ , for  $a \ll 1$ ) to  $\Omega_D(a; \nu) > 0$  at present ( $\Omega_D \rightarrow \Omega_D^0 > 0$ , for  $a \rightarrow 1$ ). This transition is not possible for  $\nu < 0$ . The divergence of  $w_e$  for positive  $\nu$  is not a real singularity of the theory since the fundamental cosmological functions in both pictures,  $\rho_M = \rho_M(a)$ ,  $\rho_\Lambda = \rho_\Lambda(a)$ ,  $\rho_D = \rho_D(a)$ ,  $H = H(a)$ , etc., are finite for all  $a$ . The discontinuity of  $w_e = w_e(a)$  is associated with the effective description of the



**Figure 2:** F-test for the running  $\Lambda$ CDM model under the same conditions as in Fig. 1. We show the factor  $F$ , Eq. (2.21), versus  $\nu$ . This parameter becomes confined in the range  $|\nu| \lesssim 0.05 \nu_0 \simeq 10^{-3}$ , due to the condition  $|F| < 0.1$ , defined in (2.23). The horizontal dashed line corresponds to the maximum allowed deviation,  $|F| = 0.1$ , of the bias factor of this running model with respect to that of the standard  $\Lambda$ CDM model.

original model in the DE picture and has no effect on our analysis. Indeed, the product function  $w_e(a) r(a)$  in the differential equations (2.16) and (2.19) remains finite for all  $a$ . Therefore, the computation of the growth factor is free from singularities in the entire range of definition.

Following the procedure explained in detail in section 2, we have determined the growth factor for both the standard  $\Lambda$ CDM and running  $\Lambda$ CDM models. For the former, we naturally used  $w_e = -1$ , and for the latter, Eq. (3.7). After solving the perturbation equation (2.19) numerically, we computed the parameter  $F$ , defined in (2.21), and applied the F-test (2.23), estimating the range of allowed values for the parameter  $\nu$ , shown in Fig. 2. The two values of  $\nu$  which saturate the F-test are the following: the upper bound,  $F_{\max} = 0.1$ , which corresponds to  $\nu_{\max} \simeq -1.3 \times 10^{-3}$  and the lower bound,  $F_{\min} = -0.1$ , which corresponds to  $\nu_{\min} \simeq +1.4 \times 10^{-3}$ . They are represented by the two black circles in Fig. 2. Therefore, the parameter  $\nu$  is confined to the range  $|\nu| \lesssim 0.05 \nu_0 \simeq 10^{-3}$ , corresponding to the shaded areas in Figs. 1 and 2. The other values,  $\nu = +0.1 \nu_0$  and  $\nu = -0.1 \nu_0$ , considered in Fig. 1, correspond to  $F_{\min} \simeq -0.17$  and  $F_{\max} \simeq 0.20$ , respectively, and are excluded because they are out of the allowed limits (the shaded region).

According to the 2dFGRS collaboration, the central values of the normalized matter and DE densities at present, obtained from the combined LSS and CMB data for



the flat  $\Lambda$ CDM Universe, are  $\Omega_M^0 = 0.27$  and  $\Omega_\Lambda^0 = 0.73$  respectively [22]. These results are in good agreement with the most recent analysis by the WMAP team, which reported  $\Omega_M^0 = 0.28$  and  $\Omega_\Lambda^0 = 0.72$  [6]. To calculate the limits on  $\nu$  in this work, we considered flat space with the following values of the matter and DE densities at present:  $\Omega_M^0 = 0.3$  and  $\Omega_D^0 = 0.7$ , respectively.

We end this section by noting that the limits we obtained for the fundamental parameter of the running  $\Lambda$ CDM model,

$$-10^{-3} \lesssim \nu \lesssim +10^{-3}, \quad (3.9)$$

are essentially in agreement with previous estimates obtained by a number of different methods and authors. In particular, the results of [31] are in good agreement with (3.9), although in the latter study an indirect procedure was employed, which was based on bounding the amplification of the matter density spectrum in the recombination era caused by vacuum decay into CDM. We emphasize that our results are also in good agreement with the full calculation of coupled perturbations of matter, metric, and DE for the running  $\Lambda$ CDM model, presented in [25], where the DE fluctuations were included as perturbations in the  $\Lambda$  parameter. A tighter bound for  $\nu$  was obtained,  $|\nu| \lesssim 10^{-4}$ . However, taking into account the economy of our present procedure, as compared to [25], we can assert that the effective approach to the calculation of the matter density fluctuations, in combination with the F-test, provides a reliable estimate of the physical region of the parameter space.

From the tiny range of values of  $\nu$  obtained, (3.9), it is patent that a procedure based solely on the search for non-standard features in the EOS profile (i.e. departures from the featureless behavior  $w_e = -1$ ) can only be efficient if sufficiently large redshift observations are used, which may be able to be obtained from the DES and SNAP programs [35]. Even these programs may have difficulty in differentiating the running  $\Lambda$ CDM model from the standard  $\Lambda$ CDM model, where  $w_e = -1.0$ . From Fig. 1, we see that the allowed interval for the effective EOS parameter of this model at the redshift  $z = 1.7$  (the largest one reachable by SNAP) is rather small, and  $w_e$  does not deviate appreciably from -1.0:

$$-1.03 \lesssim w_e(z = 1.7) \lesssim -0.97. \quad (3.10)$$

The effective EOS parameter for the running  $\Lambda$ CDM model varies very slowly with  $z$ . For example, at  $z = 2$ ,  $-1.05 \lesssim w_e(z = 2) \lesssim -0.96$ . If a small departure from the strictly constant  $w_e = -1$  could be observationally substantiated, it would be a strong

sign of dynamical behavior of the DE. A remarkable feature of the running  $\Lambda$ CDM model is that its effective EOS can mimic departures from the standard cosmological model, both in the quintessence and phantom regimes, even though the original model has nothing at all to do with quintessence or with phantom-like fields. The original model (in the “CC picture”) is nothing more or less than a model with a true, albeit running, cosmological term and, hence, with  $w_\Lambda = -1$ .

The most important result of our investigations of the running  $\Lambda$ CDM model is that the LSS data are strongly sensitive to the running of the  $\Lambda$  term, which is, of course, the reason why the parameter  $\nu$  becomes strongly constrained. This confirms the results of [25] within the present effective approach. However, as noted earlier, in this model, the variability of  $\Lambda$  is entangled with the non-conservation of matter. Therefore, in the next section we investigate a model (the  $\Lambda$ XCDM model) with a dynamical  $\Lambda$  where matter is conserved, to see if the limits on  $\nu$  can be more relaxed. In the light of the successful application of the effective method, we apply the same approach to the  $\Lambda$ XCDM model.

## 4. The framework of the $\Lambda$ XCDM model

In the introduction, we have already explained the basic motivation for the investigation of the  $\Lambda$ XCDM model, i.e., the possibility of its being a solution to the cosmic coincidence problem<sup>4</sup>. To achieve this important property, the DE density in this model must have two components, to wit: the running  $\rho_\Lambda(a; \nu)$ , which satisfies the RG evolution equation (3.1), and the dynamical component  $X$  (the “cosmon”), whose underlying nature can be very general and need not be specified here. These two components of the DE may be in interaction. As already stated,  $X$  is not necessarily related to any scalar field. It could be some effective entity, related to the structure of a more general theory, in which Einstein’s gravity is embedded, and which includes the effect of higher order terms. It is supposed that the  $X$  component behaves as a barotropic fluid with a constant EOS parameter  $w_X = p_X/\rho_X$ . (In general,  $w_X$  may not be a constant, but a function of redshift.) Typically, the index  $w_X$  for the cosmon is in one of the following two expected ranges:  $w_X \gtrsim -1$  (quintessence-like cosmon) or  $w_X \lesssim -1$  (phantom-like cosmon)<sup>5</sup>. Adopting the simplest possible  $\Lambda$ XCDM scenario,

---

<sup>4</sup>We refer the reader to a detailed discussion of this model in [18, 20].

<sup>5</sup>As noted in [18], quintessence-like and phantom-like cosmons do not necessarily exhibit the naively expected behaviors corresponding to quintessence ( $d\rho_X/da < 0$ ) and phantom energy ( $d\rho_X/da > 0$ ), respectively, since in general there is an interaction between  $X$  and the running  $\Lambda$ , see (4.2).

we assume that the total matter density does *not* interact with the DE and that it is covariantly conserved, thus satisfying (2.5). If we define the total DE density as the sum of the CC density and the cosmon density,

$$\rho_D(a) = \rho_\Lambda(a) + \rho_X(a), \quad (4.1)$$

it follows from our assumption of matter conservation that  $\rho_D$  is also covariantly conserved. The cosmon X is actually defined through this conservation condition [18]. Therefore, the quantity (4.1) satisfies Eq.(2.6), which can be recast as

$$\rho'_\Lambda(a) + \rho'_X(a) + \frac{3}{a}(1 + w_X) \rho_X(a) = 0. \quad (4.2)$$

From (4.2), it is clear that, although the total DE density is locally conserved in the  $\Lambda$ XCDM model, in general, the individual  $\rho_X$  and  $\rho_\Lambda$  densities are not. There is a transfer of energy between them, which is governed by the above equation.

The expansion history of the  $\Lambda$ XCDM model is determined by its Hubble function

$$H^2(a) = H_0^2 [\Omega_M^0 a^{-3} + \Omega_D(a)], \quad (4.3)$$

where  $\Omega_D(a) = \rho_D(a)/\rho_c^0$  is the normalized total DE density (4.1). The corresponding expression that satisfies the above equations in the matter-dominated, flat Universe is given by

$$\Omega_D(a) = \frac{\Omega_\Lambda^0 - \nu}{1 - \nu} + \frac{\epsilon \Omega_M^0 a^{-3}}{w_X - \epsilon} + \left[ \frac{1 - \Omega_\Lambda^0}{1 - \nu} - \frac{w_X \Omega_M^0}{w_X - \epsilon} \right] a^{-3(1+w_X-\epsilon)}, \quad (4.4)$$

where, for convenience, we have defined

$$\epsilon \equiv \nu(1 + w_X). \quad (4.5)$$

We will see, below, that this quantity must remain small for the model to be compatible with primordial nucleosynthesis.

The normalized densities, at present, satisfy the relation (2.1). For  $a = 1$  (i.e.  $z = 0$ ), it takes the form

$$\Omega_D^0 + \Omega_M^0 = \Omega_X^0 + \Omega_\Lambda^0 + \Omega_M^0 = 1, \quad (4.6)$$

which is the current cosmic sum rule. With the help of this relation, it is easy to see that, for  $\nu = 0$ , the DE density (4.4) boils down to

$$\Omega_D(a) = \Omega_\Lambda^0 + \Omega_X^0 a^{-3(1+w_X)}. \quad (4.7)$$

Clearly, in this particular case, where  $\nu=0$ , the  $\Lambda$ XCDM model mimics a system, consisting of a quintessence or phantom fluid (depending on the value of  $w_X$ ), together with a constant cosmological term. From (4.7), we then have three possible scenarios:

- i) If  $w_X \gtrsim -1$  and  $\Omega_X^0 > 0$  (quintessence-like cosmon), the expansion of the Universe can be stopped, provided that  $\Omega_\Lambda^0 < 0$ , since the  $X$  density gradually diminishes with the cosmic time and the *r.h.s.* of (4.7) becomes negative. Hence, there exists a future time,  $a = a_* > 1$ , when  $H(a_*) = 0$ . As mentioned previously, in the  $\Lambda$ XCDM model, the total  $\Omega_D^0 \simeq 0.7$  (corresponding to  $\Omega_M^0 \simeq 0.3$ ). Therefore, due to the contribution from  $\Omega_X^0$  in the sum rule (4.6), the possibility that  $\Omega_\Lambda^0 < 0$  should not be discarded a priori;
- ii) If  $w_X \lesssim -1$  and  $\Omega_X^0 > 0$  (phantom-like cosmon), the DE density increases indefinitely with the expansion. It does not matter whether  $\Omega_\Lambda^0$  is positive or negative, the Universe unavoidably ends up in a super-accelerated phase, which triggers a catastrophic disruption of all bound systems, a singularity known as the “Big Rip” [33];
- iii) If  $w_X \lesssim -1$  and  $\Omega_X^0 < 0$ , the cosmon density, although phantom-like, acts with matter to decelerate the expansion of the Universe. We have the opposite situation to the Big Rip: the Universe becomes super-decelerated. The kind of cosmon able to create this scenario was previously called “phantom matter” [18]. Phantom matter, therefore, avoids the Big Rip and helps the Universe to reach  $a = a_* > 1$ , where  $H(a_*) = 0$  (i.e., a stopping point). In the present instance, this point will exist provided  $\Omega_\Lambda^0 > 0$ . Obviously phantom matter is special in that it corresponds to negative energy density, which is, however, not new in the literature [34]. In spite of its rather peculiar nature, phantom matter satisfies the strong energy condition (see Fig. 1 of [18]). As previously discussed, the cosmon may well be an effective entity and, therefore, could simulate the behavior of phantom matter. This is in contrast to the “standard phantom energy”, considered in the previous case, which violates all of the classical energy conditions and leads to a cosmic doomsday.

From the previous examples, with  $\nu = 0$ , it is clear that there are simple scenarios within the  $\Lambda$ XCDM model, in which the cosmological expansion can be stopped at some point in the future. The “stopping” point is actually a “turning point” in the evolution of the Universe; it bounces back at that point and is subsequently redirected towards the Big Crunch. However, stopping can be formulated on very general grounds within the parameter space of the  $\Lambda$ XCDM model and is not restricted to  $\nu = 0$ , as in the previous examples. This issue is central to the cosmic coincidence problem [18, 19] and is correlated with the existence of a maximum of the ratio  $r(a)$ , defined in (2.15),

which gives the amount of DE energy versus matter at any time. To further address this problem, let us compute explicitly the function  $r(a)$  in the  $\Lambda$ XCDM model in the matter dominated era. With the help of (4.4) and (2.10), we find

$$r(a) = \frac{(\Omega_\Lambda^0 - \nu) a^3}{(1 - \nu) \Omega_M^0} + \frac{\epsilon}{(w_X - \epsilon)} + \left[ \frac{1 - \Omega_\Lambda^0}{\Omega_M^0 (1 - \nu)} - \frac{w_X}{w_X - \epsilon} \right] a^{-3(w_X - \epsilon)}. \quad (4.8)$$

In order to provide an acceptable explanation for the cosmic coincidence problem, this ratio should be bounded and stay relatively small throughout the entire history of the Universe. This can be expressed through the condition

$$\frac{r(a)}{r_0} < 10, \quad (4.9)$$

where  $r_0 = \Omega_D^0 / \Omega_M^0 \simeq 7/3$  is the present value of  $r$  (of order one). The ratio (4.8) should, therefore, reach a finite maximum in the evolution of the Universe. The conditions for this to occur can be expressed as [18]

$$\frac{\Omega_\Lambda^0 - \nu}{w_X (\Omega_X^0 + \nu \Omega_M^0) - \epsilon (1 - \Omega_\Lambda^0)} > 0, \quad (1 + w_X) (\Omega_\Lambda^0 - \nu) < 0. \quad (4.10)$$

We can easily check that the simple stopping scenarios i) and iii), mentioned above, are consistent with these requirements. We note that  $w_X = -1$  (i.e. a pure CC-like cosmon) is not allowed.

Besides the two conditions (4.9) and (4.10), the ratio  $r(a)$  should satisfy the nucleosynthesis constraint, namely that its value at the primordial nucleosynthesis epoch should be  $|r_N| \lesssim 0.1$  [27]. In that early epoch, the ratio  $r(a)$  is no longer given by (4.8) since we must use the radiation equation for the (relativistic) matter density, namely  $\Omega_R(a) = \Omega_R^0 a^{-4}$  instead of (2.10). Thus, we have:

$$r_N = -\frac{\epsilon}{w_R - w_X + \epsilon} + \left[ \frac{1 - \Omega_\Lambda^0}{\Omega_R^0 (1 - \nu)} - \frac{w_R - w_X}{w_R - w_X + \epsilon} \right] a_N^{-3(w_X - \epsilon) + 1}, \quad (4.11)$$

where  $w_R = 1/3$  is the barotropic index for radiation and  $a_N \sim 10^{-9}$ , the scale factor at the nucleosynthesis time. Since  $w_X < -1/3$ , the condition

$$|\epsilon| \lesssim 0.1 \quad (4.12)$$

insures that the contribution from the second term on the *r.h.s.* of (4.11) is negligible and we find that (4.12) is essentially equivalent to  $|r_N| \lesssim 0.1$ . Hereafter, we shall refer to (4.12) as the nucleosynthesis bound.

For  $\nu = 0$  and  $\Omega_X^0 = 0$  ( $\Omega_\Lambda^0 = 1 - \Omega_M^0$ ), Eq. (4.8) reduces to

$$r(a) = \frac{\Omega_\Lambda^0}{\Omega_M^0} a^3, \quad (4.13)$$

which is, of course, the standard  $\Lambda$ CDM model prediction for the ratio  $r(a)$ . We see that, as the time passes,  $a \rightarrow \infty$  and, therefore, this ratio is unbounded from above, i.e., it can take any arbitrarily large value. Thus, the cosmic coincidence problem boils down to understanding why, at  $a = 1$  (the present time), the ratio is just of order one. In other words, what makes our time special? This question has no reasonable answer within the standard  $\Lambda$ CDM model. It also has no acceptable answer within the running  $\Lambda$ CDM model. Moreover, the ratio  $r(a)$  in the running  $\Lambda$ CDM model *cannot* be worked out as a particular case of the  $\Lambda$ XCDM model because matter is not conserved in the former, whereas it is conserved in the latter.

Let us further elaborate on this ratio by considering its evaluation within the running  $\Lambda$ CDM for the two cosmological pictures that we are considering, namely the CC and the DE pictures. In the former, we have  $r(a) = \rho_\Lambda(a)/\rho_M(a)$ , where  $\rho_M(a)$  and  $\rho_\Lambda(a)$  can be obtained from equations (7)-(9) of Ref. [8], for example. The final result reads

$$r(a)_{\text{CC}} = \frac{\Omega_\Lambda^0}{\Omega_M^0} a^{3(1-\nu)} + \frac{\nu}{1-\nu} [1 - a^{3(1-\nu)}] = \frac{\Omega_\Lambda^0 - \nu}{\Omega_M^0(1-\nu)} a^{3(1-\nu)} + \frac{\nu}{1-\nu}. \quad (4.14)$$

For  $\Omega_\Lambda^0 < \nu$ , the CC density eventually becomes negative and, in that case, there is stopping in the running  $\Lambda$ CDM. However, since  $\Omega_\Lambda^0 \simeq 0.7$ , this possibility entails a value of  $\nu$ , which is ruled out by our result (3.9). Hence, there is no viable solution to the cosmic coincidence problem in this model. We wish to stress that this result is independent of the particular cosmological picture chosen to derive it. Indeed, in the DE picture the ratio  $r(a)$  is, instead,  $r(a) = \rho_D(a)/\rho_M(a) = \Omega_D(a)/\Omega_M(a)$ , where  $\Omega_M(a)$  and  $\Omega_D(a)$  are given by (2.10) and (3.6) respectively. Therefore,

$$r(a)_{\text{DE}} = \left( \frac{\Omega_\Lambda^0}{\Omega_M^0} - \frac{\nu}{1-\nu} \right) a^3 + \frac{a^{3\nu} - 1 + \nu}{1-\nu} = \frac{\Omega_\Lambda^0 - \nu}{\Omega_M^0(1-\nu)} a^3 + \frac{a^{3\nu}}{1-\nu} - 1, \quad (4.15)$$

where we used  $\Omega_M^0 = 1 - \Omega_\Lambda^0$ . Notice that both ratios,  $r(a)_{\text{DE}}$  and  $r(a)_{\text{CC}}$ , satisfy the correct normalization at the present time, i.e.,  $r(a=1) = \Omega_\Lambda^0/\Omega_M^0$ . Again, we see from (4.15) that the condition for the DE density to become increasingly negative (leading to stopping) will occur only if  $\Omega_\Lambda^0 < \nu$ . Thus, we obtain the same conclusion as in the CC picture.

The foregoing results indeed show that irrespective of the cosmological picture used to perform the analysis, the conclusion is the same, to wit: in the running  $\Lambda$ CDM model there is no natural solution to the cosmic coincidence problem. Although this model does provide some interesting dynamics for the CC term, it does not have the ability to ameliorate the cosmic coincidence problem. It is only when the X entity is

introduced in interplay with a dynamical  $\Lambda$ , that the ratio  $r(a)$  takes the form of (4.8) and can be kept within bounds throughout the entire history of the Universe, which is indeed what is needed to solve the cosmic coincidence problem.

The three conditions, (4.9), (4.10) and (4.12), define a limited 3D-region in the parameter space  $(\Omega_\Lambda^0, w_X, \nu)$  of the  $\Lambda$ XCDM model at a fixed  $\Omega_M^0 \simeq 0.3$  [18]. In the next section, we give a detailed numerical analysis of the  $\Lambda$ XCDM model, after including two more conditions, both of which are related to the effective EOS of the model. One of the conditions is its compatibility with the LSS data. We express this compatibility condition, using the F-test (2.23) on the linear bias parameter of the model, which depends on the effective EOS approach to the calculation of the growth factor. The other condition is the maximum allowed deviation that we can tolerate for the value of the effective EOS parameter,  $w_e = w_e(z)$ , away from the CC value,  $w_e = -1$ , at  $z = 0$ . As we shall see, with these five conditions, we will be able to significantly improve the determination of the physical volume in the 3D parameter space of the  $\Lambda$ XCDM model, as compared to [18].

## 5. Numerical analysis of the $\Lambda$ XCDM model: cosmic matter perturbations versus cosmic coincidence

In this section, we present a complete numerical analysis of the  $\Lambda$ XCDM model, in which its most salient feature is the inclusion of the effective EOS approach to the growth of matter density fluctuations, together with its ability to solve (or at least significantly alleviate) the problem of cosmic coincidence, mentioned in the introduction. The effective EOS for this model is [18]

$$w_e(a) = \frac{p_\Lambda + p_X}{\rho_\Lambda + \rho_X} = \frac{-\rho_\Lambda + w_X \rho_X}{\rho_\Lambda + \rho_X} = -1 + (1 + w_X) \frac{\Omega_X(a)}{\Omega_D(a)}, \quad (5.1)$$

where  $\Omega_D = \Omega_D(a)$  is given by (4.4), and

$$\Omega_X(a) = \left( \Omega_X^0 - \frac{\nu}{w_X - \epsilon} \Omega_M^0 \right) a^{-3(1+w_X-\epsilon)} + \frac{\nu}{w_X - \epsilon} \Omega_M^0 a^{-3}. \quad (5.2)$$

Note that for  $\nu = 0$ , Eq. (5.2) reduces to the expression in the second term on the *r.h.s.* of (4.7), as it should. Equations (5.2) and (4.4) may be used to compute the effective EOS (5.1). Alternatively, (4.4) may be substituted in (2.9), the two procedures being equivalent since

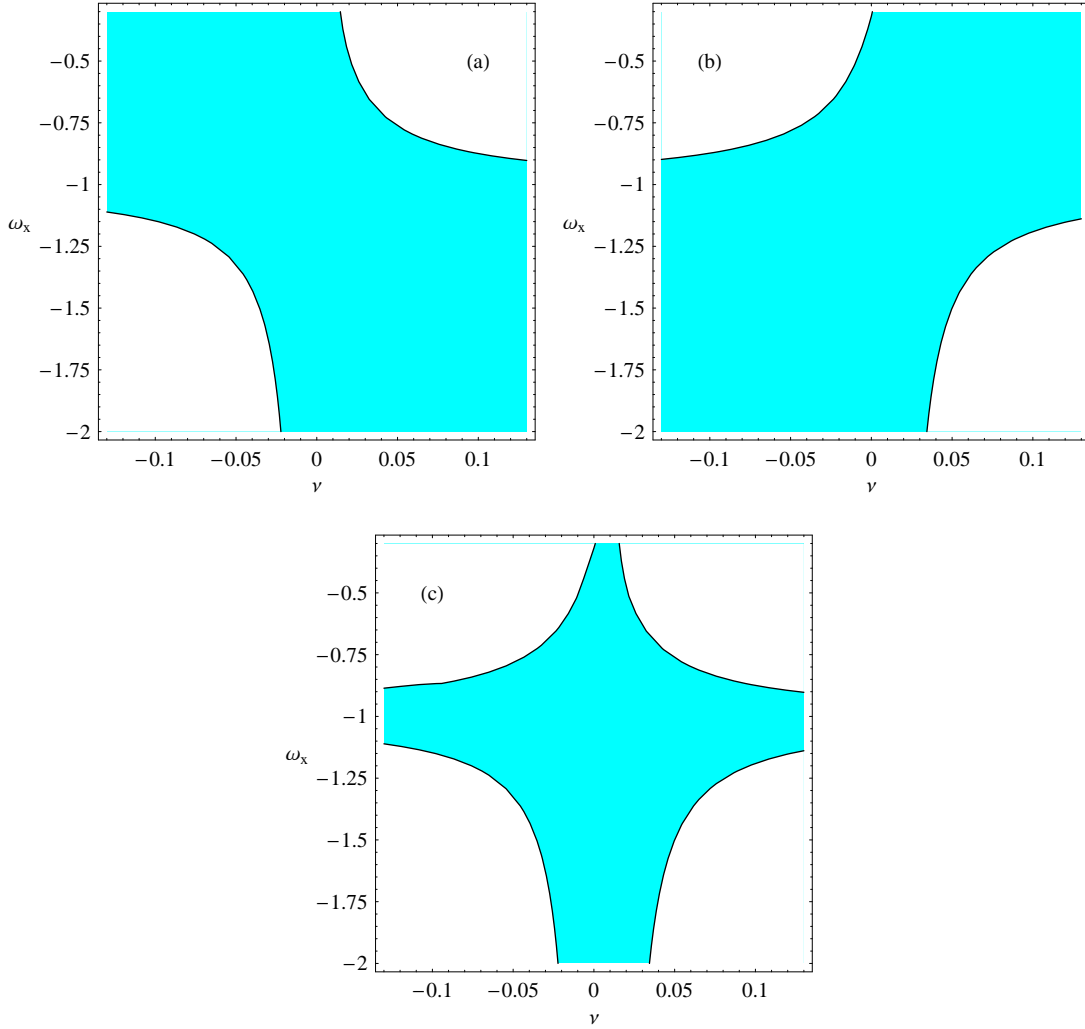
$$(1 + w_X) \Omega_X(a) = -\frac{a}{3} \frac{d\Omega_D(a)}{da}. \quad (5.3)$$

Clearly, the dynamical features of the cosmon play a preeminent role in the behavior of the effective EOS for the  $\Lambda$ XCDM model. For example, if the cosmon is quintessence-like ( $w_X \gtrsim -1$  and  $\Omega_X(a) > 0$ ) near  $a = 1$ , the overall effective EOS of the model will be quintessence-like near our time. However, if the cosmon behaves as phantom matter (viz.  $w_X \lesssim -1$  and  $\Omega_X(a) < 0$ ), the effective EOS of the model will still be quintessence-like. On the other hand, if the cosmon behaves as standard phantom DE ( $w_X \lesssim -1$  and  $\Omega_X(a) > 0$ ), the  $\Lambda$ XCDM model will also behave phantom-like as a whole. In general, the  $\Lambda$ XCDM model behaves effectively as quintessence (phantom) near  $z = 0$ , if and only if  $1 + w_X$  and  $\Omega_X$  have the same (opposite) signs. This can be seen from (5.3) and (5.1).

For the numerical analysis, we insert the EOS formula (5.1) into (2.19) for the effective growth of the density perturbations. We then solve this equation and perform the F-test (2.23). That is, we impose a condition on structure formation, whereby we discard all points of the parameter space, for which the growth factor of our model at  $z = 0$  deviates by more than 10% from that of the  $\Lambda$ CDM. In the analysis, we have to also include the three conditions discussed at the end of the last section, namely, the primordial nucleosynthesis constraint, the stopping condition, and the bounding condition on the ratio  $r = r(a)$ . We begin the analysis by checking the ability of the F-test, alone, (2.23), to define a limited region of the  $(w_X, \nu)$  plane. As mentioned previously, we assume a flat Universe with  $\Omega_M^0 = 0.3$  and  $\Omega_D^0 = 0.7$ . For the sake of illustration, we split the DE density into its individual components as follows:  $\Omega_\Lambda^0 = 0.65$  and  $\Omega_X^0 = 0.05$  (which, of course, add up to the value of  $\Omega_D^0$ ). In Fig. 3, we show the regions of  $(w_X, \nu)$  plane that fulfill the partial constraints:  $F < 0.1$  (Fig. 3a) and  $F > -0.1$  (Fig. 3b), as well as the more restrictive region  $|F| < 0.1$  (Fig. 3c), which represents their intersection. All points in these figures automatically satisfy the nucleosynthesis bound (4.12). This means that, in this case, the F-test is already more restrictive than the nucleosynthesis bound.

We note that  $w_X$  is less constrained when  $\nu = 0$ , i.e., when the  $\Lambda$ XCDM model behaves as a quintessence (or phantom) model with a cosmological constant – see (4.7). In this case, we have  $-14 \lesssim w_X \lesssim -0.3$ . Similarly, when  $w_X$  is very close to the value  $w_X = -1$ , the cosmon behaves as a cosmological constant and the range of  $\nu$  is almost unconstrained by the F-test. This scenario effectively corresponds to having a strict cosmological constant (in this case, the cosmon), together with a variable cosmological term,  $\Lambda$ . These unconstrained situations will change dramatically when the other restrictions (particularly that of *non-cosmic coincidence*) are also imposed





**Figure 3:** F-test condition (2.23) for the  $\Lambda$ XCDM model assuming flat geometry with  $\Omega_M^0 = 0.3$  and individual DE densities  $\Omega_\Lambda^0 = 0.65$  and  $\Omega_X^0 = 0.05$ . Shown are: a) region of the  $(\nu, w_X)$  plane in which  $F < 0.1$ ; b) the corresponding region where  $F > -0.1$ ; and c) the allowed region by the F-test ( $|F| < 0.1$ ), i.e. the intersection of regions a) and b) .

in combination with the F-test. We have already seen, in the previous section, that  $w_X = -1$  is actually forbidden. However, the F-test, alone, (as a strategy to determine the principal restrictions due to structure formation) is able to highly constrain regions, which the other conditions are not able to do, as we will see below.

By assuming that  $\Omega_M^0 = 0.3$  for the present matter content of the Universe (which can be deduced from LSS observations alone) and that the Universe is flat, the  $\Lambda$ XCDM model is left with three free parameters, namely,  $w_X$ ,  $\nu$  and  $\Omega_\Lambda^0$ . The cosmon density  $\Omega_X^0$  is, then, no longer independent, since it is fixed by the cosmic sum rule (4.6). In a previous work [18], the parameter space of the  $\Lambda$ XCDM model was constrained by

imposing the three conditions (4.9), (4.10), and (4.12), emerging from three important physical requirements:

- *Condition 1: Nucleosynthesis bound.*

The expansion rate depends directly on the amount of DE. If we are not to spoil the standard Big Bang predictions (about, e.g., light element abundances), the DE density at the nucleosynthesis time should not be very large. Specifically, it is required that [27]

$$\tilde{\Omega}_D(a_N) = \frac{r_N}{1 + r_N} \lesssim 0.1, \quad (5.4)$$

which is essentially equivalent to

$$|r_N| \equiv \left| \frac{\rho_D(z)}{\rho_M(z)} \right|_{(z=z_N)} \lesssim 0.1, \quad (5.5)$$

where  $z_N \sim 10^9$  is the cosmological redshift at the nucleosynthesis era.

- *Condition 2: Stopping condition.*

As commented in section 4, in the  $\Lambda$ XCDM model, the ratio  $r(a)$  of the DE to matter density may exhibit a maximum. This feature is related to a future stopping and subsequent reversal of the expansion of the Universe, expressed by (4.10).

- *Condition 3: Low maximum of the ratio  $r(a)$ .*

For a solution, or at least a substantial alleviation, of the coincidence problem, we must further require that the stopping point of the expansion of the Universe is preceded by a sufficiently small maximum value of the ratio  $r(a)$ , defined in (2.15). In the standard  $\Lambda$ CDM model,  $r(a)$  is unbounded and its present value,  $r_0 \sim 1$ , is related to the recent transition from decelerated to accelerated expansion. Thus, within the standard  $\Lambda$ CDM, we can conclude that we are living in a very special epoch, i.e., very close to the transition epoch. Alternatively, if  $r(a)$  remains bounded and sufficiently small for the entire history of the Universe, as can be the case in the  $\Lambda$ XCDM model, the fact that  $r_0 \sim 1$  should no longer be regarded as a coincidence since the relation  $r \sim 1$  could hold for most of the lifetime of the Universe. Therefore, in order to accommodate this appealing feature in the  $\Lambda$ XCDM model, we search for points in the parameter space that not only allow for the existence of a maximum for  $r(a)$ , but also those points, for which the value of the maximum is sufficiently small. Specifically, we express quantitatively this condition by (4.9).

Conditions 2 and 3 are actually related since the existence of a maximum for  $r(a)$  is correlated with the existence of stopping. We have, nevertheless, kept them separated in order to stress that the stopping condition is not sufficient to solve the cosmic coincidence problem, it is only a necessary condition. We still have to demand a “qualified stopping”, i.e., one preceded by a sufficiently low maximum. Conditions 2 and 3 could then be unified and be collectively referred to as *The ability to solve the cosmic coincidence problem*.

The subset of points satisfying these three conditions was found in [18] to constitute a volume in the 3D parameter space, shown in Fig. 4 (upper panel). In this paper, we have incorporated a fourth and very important condition in the numerical analysis:

- *Condition 4: Consistency with the data on structure formation.*

We have already explained in detail, throughout this work, the way in which we have included this condition in our analysis, namely, through the effective EOS approach and the implementation of the F-test, (2.23). This has been one of the principal aims of this work.

There is yet one more observational requirement that can be demanded:

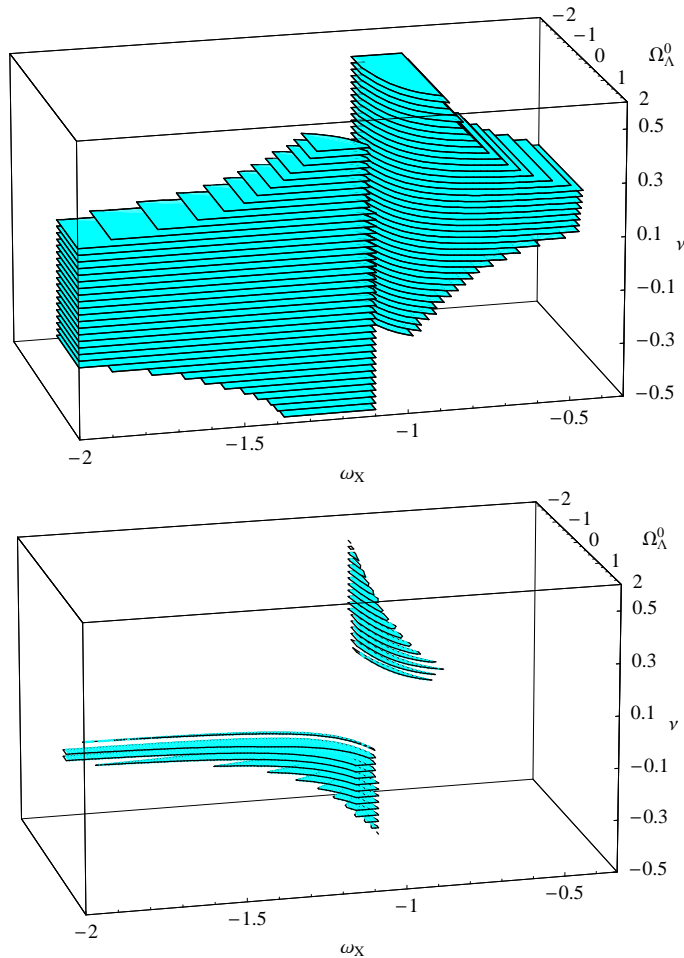
- *Condition 5: EOS condition at  $z = 0$*

The value of the effective EOS parameter should behave as a CC in the recent past. However, the effective EOS of the  $\Lambda$ XCDM model (5.1) does *not* automatically satisfy the “CC condition”,  $w_e(z) \rightarrow -1$  for  $z \rightarrow 0$ , as does the effective EOS of the running  $\Lambda$ CDM model (3.7). Therefore, we wish to make sure that this condition is indeed satisfied by the  $\Lambda$ XCDM model, within the limits of the latest observational data. We normalize this EOS bound at  $z = 0$ , which is elaborated below. The quantitative restriction associated with this condition is

$$|1 + \omega_e(z = 0)| \leq 0.3. \quad (5.6)$$

This can be justified from recent experiments, which strongly suggest that the EOS parameter should be close to  $-1$ . For instance, the combination of WMAP and the Supernova Legacy Survey (SNLS) [36] data (under the assumption of spatial flatness) yields [6]:

$$\omega_e = -0.967^{+0.073}_{-0.072}. \quad (5.7)$$



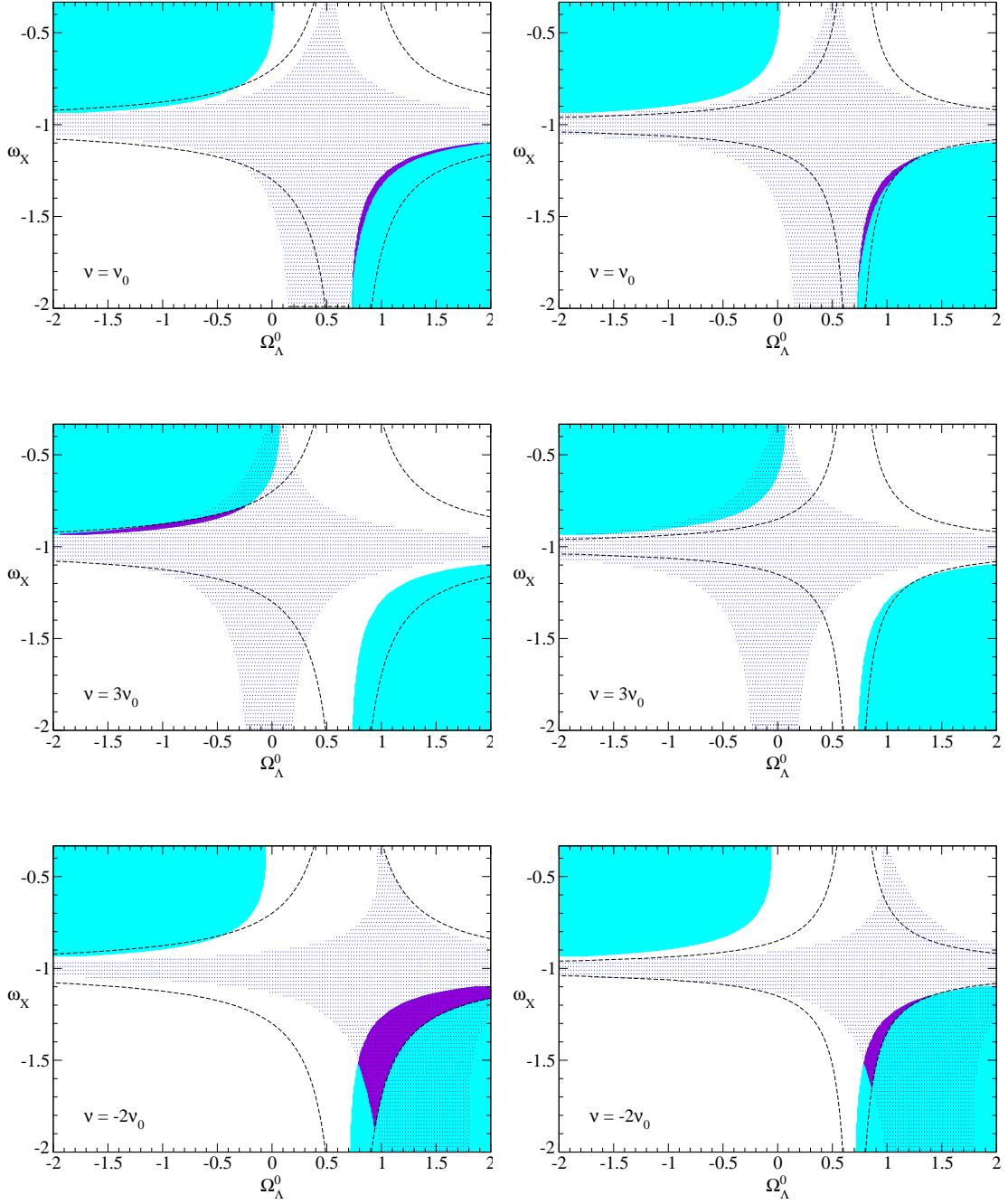
**Figure 4:** Upper panel: Volume of points allowed by *Conditions 1-3* described in the text, i.e. nucleosynthesis plus the two restrictions associated with the solution of the coincidence problem; Lower panel: Volume of points allowed by *all* constraints discussed in the text, namely *Conditions 1-5*, which include the LSS constraint and the EOS restriction at  $z = 0$ . We assume flat space geometry with  $\Omega_M^0 = 0.3$  and  $\Omega_D^0 = 0.7$ .

Even without the prior that the Universe is flat, the value of the EOS parameter preferred by WMAP, large-scale structure and supernovae data is still very close to that of a cosmological constant [6],

$$\omega_e = -1.08 \pm 0.12. \quad (5.8)$$

These observational results assume that the EOS parameter is constant and, therefore, they are not directly applicable to the  $\Lambda$ XCDM model. Nevertheless, we prefer to adopt a conservative point of view and impose the additional constraint from (5.6), which we referred to as condition 5, to our model.

Remarkably enough, after imposing the five conditions listed above, we are still



**Figure 5:**  $\nu$ -slices of the physical 3D region of the  $\Lambda$ XCDM parameter space (Fig. 4, lower panel) for  $\nu = \nu_0$ ,  $3\nu_0$  and  $-2\nu_0$ . Points in the solid-shaded area fulfil the nucleosynthesis bound and the two conditions associated with the solution of the coincidence problem (*Conditions 1,2 and 3* in the text). The dotted region consists of points allowed by the LSS data (*Condition 4*). Points inside the dashed lines satisfy *Condition 5*:  $|1+\omega_e(0)| \leq 0.3$  (left panel) and  $|1+\omega_e(0)| \leq 0.15$  (right panel). The physical region (the darker one) in each case is the common overlap. Note that quintessence-like cosmons are forbidden if  $|1+\omega_e(0)| \leq 0.15$ .

left with a non-empty volume of points in 3D parameter space that satisfy all of them. This can be seen in Fig. 4 (lower panel). Comparing with the upper panel of this figure, we see that the new constraints greatly diminish the final volume of points allowed in the parameter space. This means that conditions 4 and 5 are very restrictive when combined with the first three conditions.

If we wish to better visualize the effect of the different constraints, it is more convenient to study two-dimensional slices of the final 3D volume in Fig. 4 (lower panel). In order to do this, we fix one of the three free parameters. In Fig. 5, we plot three different  $\nu$ -slices of the final volume:  $\nu = \nu_0, 3\nu_0, -2\nu_0$ , where  $\nu_0$  is the canonical value, defined in (3.3). In Fig. 6, we plot three  $\Omega_\Lambda^0$ -slices:  $\Omega_\Lambda^0 = -0.65, 0.8, 1.2$ . Let us discuss the  $\nu$ -slices and restrict ourselves, for the moment, to the panel on the left of Fig. 5. We see that both signs of  $\nu$  are possible and that the cosmon may be both quintessence-like ( $w_X \gtrsim -1$ ) and phantom-like ( $w_X \lesssim -1$ ), although the former is only possible for positive  $\nu$ , according to Fig. 5 (middle figure on the left). This feature does not depend on this particular  $\nu$ -slice ( $\nu = 3\nu_0$ ), it is general. It can be confirmed from Fig. 7, where the projections of the bulk volume of final points (lower panel of Fig. 4) onto the three possible planes are shown. Also general is the property that phantom-like cosmons are compatible with positive (greater than +0.7) values of  $\Omega_\Lambda^0$ , whereas quintessence-like cosmons demand a negative energy density of the vacuum (cf. Fig. 7).

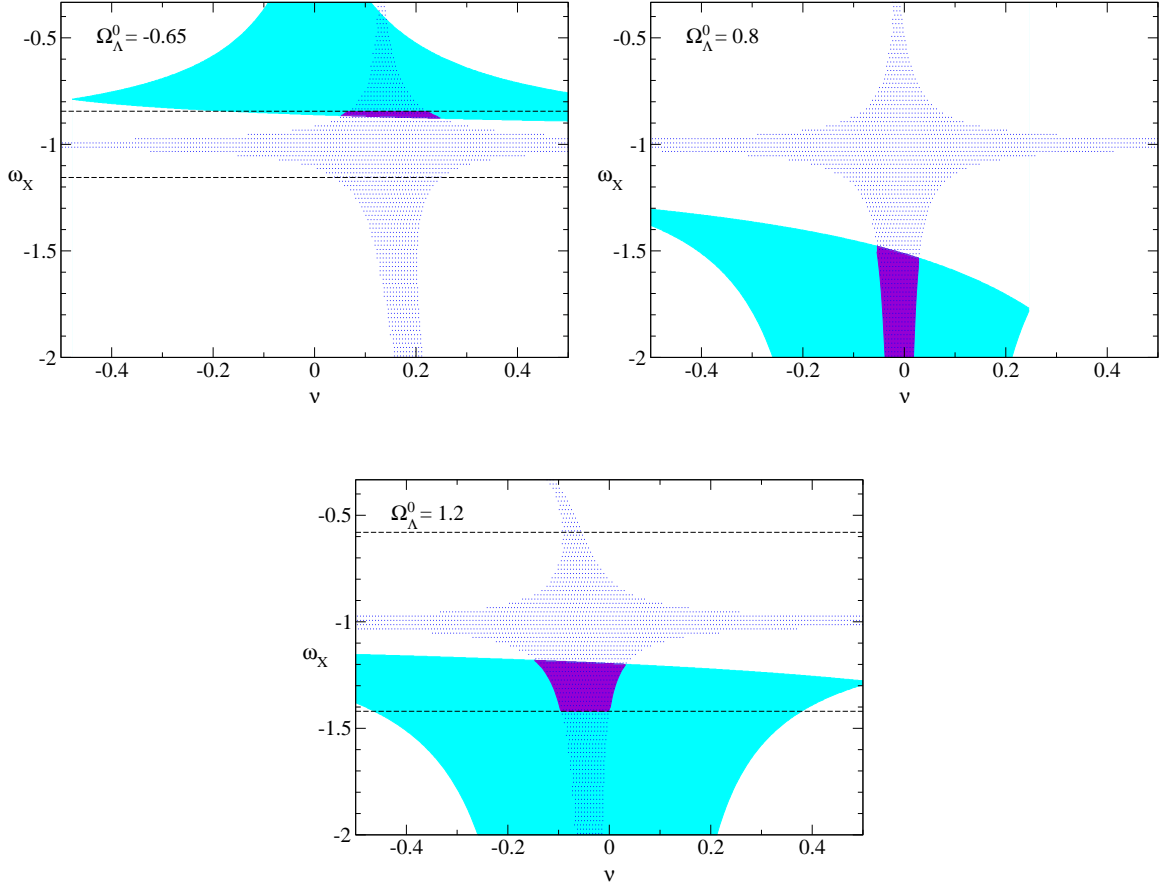
Since the precision of observational data will increase in the future, we have also studied what happens if we make the condition on  $\omega_e(0)$  more stringent than condition (5.6), namely,

$$|1 + \omega_e(0)| \leq 0.15. \quad (5.9)$$

The result of applying this tighter constraint is that the volume of points allowed becomes further reduced, as can be seen in the panel on the right of Fig. 5. The most conspicuous effect is that, now, the  $X$  component can only be phantom-like ( $w_X \lesssim -1$ ); quintessence-like cosmons ( $w_X \gtrsim -1$ ) are no longer permitted.

Consider now the  $\Omega_\Lambda^0$ -slices ( $\Omega_\Lambda^0 = -0.65, 0.8, 1.2$ ) of the final volume in Fig. 6. In general, only a small subset of all the points allowed by the F-test is allowed by the full set of constraints. Conversely, the area allowed by the old constraints (*Conditions 1, 2 and 3*) is highly restricted by the F-test. For example, the  $\Omega_\Lambda^0 = -0.65$  slice shows a critically constrained case, where the concurrence of the five conditions, particularly the EOS restriction at  $z = 0$  (*Condition 5*), leave a very small area allowed in that slice.

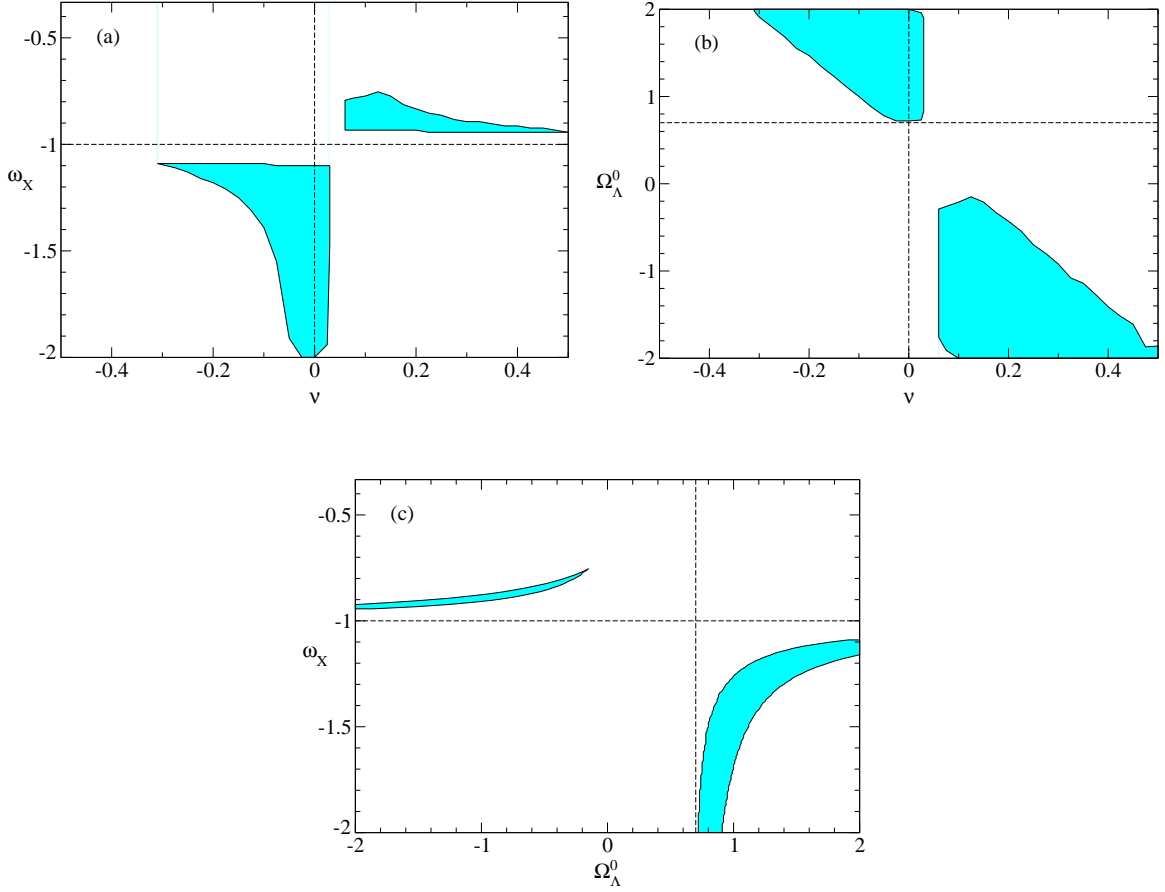
Let us now consider the projections of the final allowed volume in Fig. 7. The signs



**Figure 6:** Three  $\Omega_\Lambda^0$ -slices of the physical 3D region of the  $\Lambda$ XCDM parameter space for  $\Omega_\Lambda^0 = -0.65, 0.8, 1.2$ . The shaded region is allowed by *Conditions 1-3* in the text. The dotted one is allowed by *Condition 4* (the F-test), and the region between the vertical dashed lines is allowed by *Condition 5*. The  $\Omega_\Lambda^0 = 0.8$  plot is not restricted by this condition. The darker areas in the three plots are the final allowed regions.

of  $\nu$  and  $\Omega_\Lambda^0$  are correlated, as shown in Fig. 7b. Negative values of  $\Omega_\Lambda^0$  are compatible only with positive values of  $\nu$ . Similarly, positive values of  $\Omega_\Lambda^0$  admit mostly negative values of  $\nu$ , although there is still a range of  $\nu > 0$  allowed values, which are of order of the canonical value  $\nu_0 \sim 10^{-2}$ . From Fig. 7c, we see that the signs of  $1 + w_X$  and  $\Omega_\Lambda^0$  are also correlated. Thus, quintessence-like cosmons ( $1 + w_X \gtrsim 0$ ) demand that  $\Omega_\Lambda^0 < 0$ , whereas phantom-like ones ( $1 + w_X \lesssim 0$ ) require that  $\Omega_\Lambda^0 > +0.7$ . However, it should be stressed that if we would apply the tighter condition, indicated in (5.9), only the phantom-like character is possible for the cosmon; in other words, the thin quintessence regions in Fig. 7a and Fig. 7c then disappear altogether.

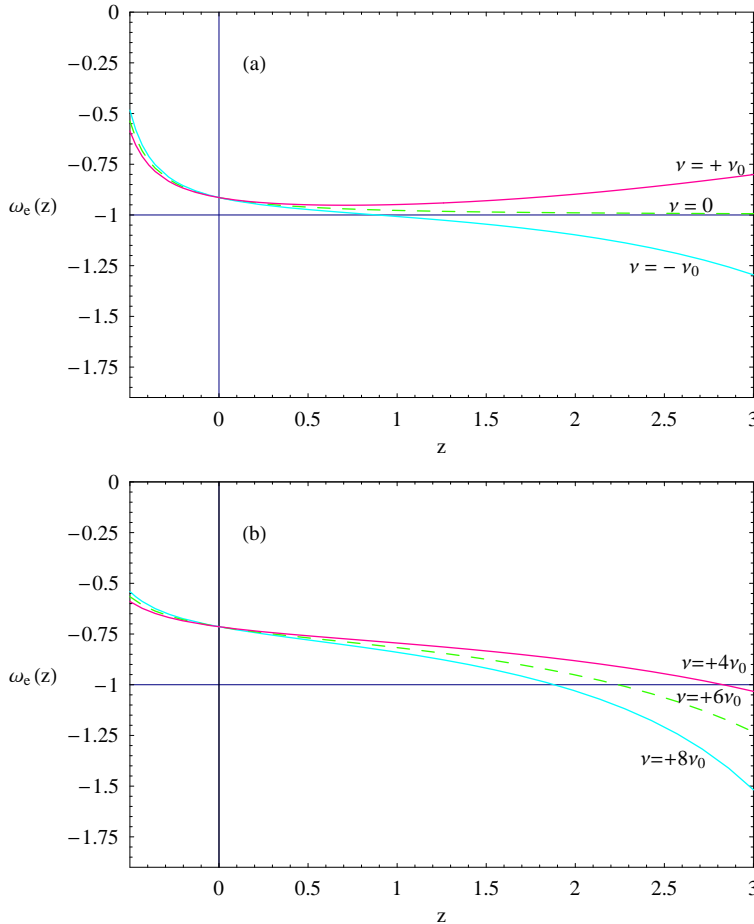
The reduction of the original region, permitted by the first three conditions, to the final region, filtered by all the five conditions, is best seen, comparing the projection



**Figure 7:** Projections of the final 3D physical region of the  $\Lambda X$ CDM model (Fig. 4, lower panel) onto the three different planes. The vertical and horizontal dashed lines mark the values  $\nu = 0$ ,  $w_X = -1$  and  $\Omega_\Lambda^0 = +0.7$  wherever any of these parameters appear.

plots in Fig. 7 with the old projection plots in Fig. 3 of [18]. For instance, the present range of values for the  $\nu$  parameter become bounded from above and below and it is no longer possible to have  $|\nu| > 0.5$ . Another dramatic restriction that occurs in the present analysis is the near exclusion of the region where the cosmon is quintessence-like, the very small region of the allowed parameter space where  $w_X \gtrsim -1$ . In this region,  $\nu$  cannot be negative or zero (or even close to zero) and must be  $\nu \gtrsim 0.1$ . This is clearly seen in the projection plot of Fig. 7a. We conclude that in most of the parameter space, the cosmon  $X$  behaves phantom-like ( $w_X \lesssim -1$ ). Since, however,  $\Omega_\Lambda^0 > +0.7$  in this region and the current value of the total DE density is fixed at  $\Omega_D^0 = +0.7$ , the cosmic sum rule (4.6) indicates that in most of the allowed parameter space,  $\Omega_X^0 < 0$ . Put another way, the cosmon behaves mostly as phantom matter. This was to be expected since phantom matter satisfies, as noted previously, the strong energy condition [18], and therefore it effectively behaves like additional matter, helping





**Figure 8:** The effective EOS (5.1) of the  $\Lambda$ XCDM model as a function of the redshift for a set of allowed values, see Fig. 7: (a)  $w_X = -1.6$ ,  $\Omega_\Lambda^0 = 0.80$  (phantom-like cosmon) for three values of  $\nu$ ; (b)  $w_X = -0.8$ ,  $\Omega_\Lambda^0 = -0.3$  (quintessence-like cosmon) for three positive values of  $\nu$  (no negative values allowed in this case)

to stop the expansion of the Universe at some time in the future.

We emphasize that this preference for the phantom matter character of the cosmon entity does not necessarily translate to the overall effective EOS of the  $\Lambda$ XCDM model. This has already been discussed after Eq. (5.3). The quintessence-like behavior ( $d\Omega_D/da < 0$ ) or phantom-like behavior ( $d\Omega_D/da > 0$ ) of the total DE of the  $\Lambda$ XCDM model depends only on the sign of the product  $(1 + w_X)\Omega_X$  in the regions where  $\Omega_D$  is positive. Let us consider some examples, particularly those in Fig. 8a, which correspond to allowed values of the parameters. Since, in this case,  $\Omega_X^0 = -0.1$  is small, from Eq. (5.2) we can see that for sufficiently large  $z$  (small  $a$ ), the last term on the *r.h.s.* is the dominant one (using the fact that  $\epsilon \ll 1$ ). Therefore, we have the following rule of thumb: the total DE of the  $\Lambda$ XCDM model behaves as quintessence

( $w_e \gtrsim -1$ ) or phantom ( $w_e \lesssim -1$ ), if the sign of  $\nu(1+w_X)/w_X$  is positive or negative, respectively:

$$1 + w_e(z) \gtrless 0 \iff \frac{\nu(1+w_X)}{w_X} \gtrless 0. \quad (5.10)$$

For example, if  $X$  behaves as phantom matter (which is the preferred state of the cosmon, according to our analysis), the  $\Lambda$ XCDM model can be phantom-like if  $\nu < 0$ , but behaves like quintessence if  $\nu > 0$  (cf. Fig. 8a). When the first term on the *r.h.s.* of Eq. (5.2) becomes important, the rule applies only if  $z$  is sufficiently large, as it is the case of the examples in Fig. 8b, where  $\Omega_X^0 = 1$ . These examples illustrate that the overall behavior of the effective EOS of the  $\Lambda$ XCDM model is not dictated by the cosmon EOS alone, but depends on other parameters, in particular  $\nu$ .

Finally, we may compare Fig. 8 of the  $\Lambda$ XCDM model with Fig. 1 of the running  $\Lambda$ CDM model, studied in the previous section. For the  $\Lambda$ XCDM, the allowed values of  $\nu$  are larger. As a result, the departure of the effective EOS of this model with respect to the CC divide,  $w_e = -1$ , can be significantly greater in the redshift ranges that are accessible to the next generation of supernovae experiments. It is, therefore, possible to investigate deviations of  $w_e$  from -1, predicted by the  $\Lambda$ XCDM model, in these experiments, particularly by SNAP [35]. For instance, in Fig. 8a we see that for  $\nu = -\nu_0$ , we have the predicted value  $w_e(z = 1.7) = -1.06$  and for  $\nu = +\nu_0$ , we have  $w_e(z = 1.7) = -0.92$ , corresponding to the highest redshift accessible to SNAP. The departure from  $-1$  is significant large to be detected. At  $z = 2$ , which will be accessible in the future, we have  $w_e = -1.10$  and  $w_e = -0.90$  for the same values of  $\nu$ .

## 6. Conclusions

In this paper, we have explored the growth of matter density perturbations for two running  $\Lambda$  models in the literature, the running  $\Lambda$ CDM model [7, 8, 11] and the  $\Lambda$ XCDM model [18, 20]. These models offer alternative explanations for the dynamical dark energy (DE), beyond the usual proposals based on quintessence ideas. In particular, the  $\Lambda$ XCDM model constitutes a promising cosmological framework, of a very general nature, that also has the capacity to try to understand the conspicuous cosmic coincidence problem, namely the perplexing coincidence of finding ourselves, at present, in an expanding Universe, where the amount of dark energy turns out to be of the same order as that of matter. The cosmic coincidence is a riddle, wrapped in the polyhedric mystery of the Cosmological Constant Problem [1], which has many faces. In this case, the conundrum is to understand the following situation. The density of

matter is continuously decreasing with the expansion of the Universe, whereas the DE energy remains constant in the standard  $\Lambda$ CDM model. It is totally inexplicable to understand from this model, the fact that the cosmological constant line crosses the ever-falling matter density curve, precisely now. The  $\Lambda$ XCDM model may be able to provide a clue to the resolution of this enigma, due to the dynamical interplay of the cosmon  $X$  and the variable  $\Lambda$ , which together, form a composite dark energy medium. This dynamics insures that the ratio of the DE to the matter density stays bounded and that its value, at present, is not very different from the value it will have in, say, the next Hubble time.

An important aspect of the  $\Lambda$ XCDM model that has been yet to be investigated is its consistency with the LSS data. In this paper, we have undertaken a thorough study, in an attempt to try to answer this crucial question, namely, is there a non-empty region in the 3D parameter space of the  $\Lambda$ XCDM model, capable of solving the cosmic coincidence problem and, at the same time, being consistent with the known data on structure formation? Our study shows that the answer to this question was, in fact, positive.

The “effective approach” that we have used here is based on three essential ingredients: i) the use of the effective equation of state (EOS) representation of cosmologies with variable cosmological parameters [12, 13]; ii) the calculation of the growth of matter density fluctuations, using the EOS of the DE [27, 28, 29]; and iii) the application of the “F-test” [30, 31] to compare the model with the LSS data. This three-step methodology turned out to be a streamlined strategy. Even if it is not a perfect procedure to estimate the restrictions that structure formation imposes on a given model of dark energy, it is nevertheless an economical and efficient method to encapsulate the essential findings of the full-fledged approach. That this is so can be argued on the grounds of the various existing cross-checks on the constraints that LSS data impose e.g. on the running  $\Lambda$ CDM model, the first dynamical  $\Lambda$  model that we addressed in this study. Our “effective approach” provides a noticeable consistency with the complete calculation of matter and DE perturbations presented for the running  $\Lambda$ CDM model in Ref. [25]. We are, therefore, confident that the same procedure is able to capture the main restrictions that the present data put on the parameter space of the  $\Lambda$ XCDM model.

In view of the consistency between the solution to the cosmic coincidence problem proposed by the  $\Lambda$ XCDM model and the data on structure formation, this model is substantially reinforced. It emerges as a very strong candidate for a possible solution

to the cosmic coincidence problem. The physical region of its parameter space turns out to be compatible with all cosmological data known at present. Furthermore, we have shown (cf. Fig. 8) that the model predicts non-trivial observable features in the EOS of the dark energy. Most important, these features can be accessible to the next generation of supernovae experiments, like DES and SNAP [35]. This model, therefore, has the ability to solve some of the important problems of modern cosmology and, in addition, makes predictions which can be tested by observations from experiments just around the corner. We eagerly await the possibility of confronting the  $\Lambda$ XCDM model with these observations.

**Acknowledgments.** JG, AP and JS are partially supported by MECYT and FEDER under project 2004-04582-C02-01, and also by DURSI Generalitat de Catalunya under project 2005SGR00564. RO is partially supported by the Brazilian Agency CNPq (Grant no.300699/2006-91).

## References

- [1] S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1; T. Padmanabhan, *Phys. Rep.* **380** (2003) 235.
- [2] See e.g. V. Sahni, A. Starobinsky, *Int. J. of Mod. Phys.* **A9** (2000) 373; S.M. Carroll, *Living Rev. Rel.* **4** (2001) 1; T. Padmanabhan, *Curr. Sci.* **88** (2005) 1057; E.J. Copeland, M. Sami, S. Tsujikawa, *Int. J. of Mod. Phys.* **D15** (2006) 1753.
- [3] A.G. Riess *et al.*, *Astronom. J.* **116** (1998) 1009; S. Perlmutter *et al.*, *Astrophys. J.* **517** (1999) 565; R. A. Knop *et al.*, *Astrophys. J.* **598** (2003) 102; A.G. Riess *et al.* *Astrophys. J.* **607** (2004) 665.
- [4] B. Ratra, P.J.E. Peebles, *Phys. Rev.* **D37** (1988) 3406; C. Wetterich, *Nucl. Phys.* **B302** (1988) 668; R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* **80** (1998) 1582.
- [5] For a review, see e.g. P.J.E. Peebles, B. Ratra, *Rev. Mod. Phys.* **75** (2003) 559, and the long list of references therein.
- [6] D.N. Spergel *et al.*, *WMAP three year results: implications for cosmology*, [astro-ph/0603449](#).
- [7] I.L. Shapiro, J. Solà, *JHEP* 0202 (2002) 006, [hep-th/0012227](#); *Phys. Lett.* **B475** (2000) 236, [hep-ph/9910462](#).
- [8] I.L. Shapiro, J. Solà, *Nucl. Phys. Proc. Supp.* **127** (2004) 71, [hep-ph/0305279](#).

- [9] I.L. Shapiro, J. Solà, H. Štefančić, *JCAP* **0501** (2005) 012, [hep-ph/0410095](#).
- [10] I.L. Shapiro, J. Solà, *J. Phys.* **A40** (2007) 6583, [gr-qc/0611055](#).
- [11] I.L. Shapiro, J. Solà, C. España-Bonet, P. Ruiz-Lapuente, *Phys. Lett.* **B574** (2003) 149; *JCAP* 0402 (2004) 006, [hep-ph/0311171](#); I. L. Shapiro, J. Solà, JHEP proc. AHEP2003/013, [astro-ph/0401015](#).
- [12] J. Solà, H. Štefančić, *Phys. Lett.* **B624** (2005) 147, [astro-ph/0505133](#).
- [13] J. Solà, H. Štefančić, *Mod. Phys. Lett.* **A21** (2006 ) 479, [astro-ph/0507110](#); *J. Phys.* **A39** (2006) 6753, [gr-qc/0601012](#).
- [14] A. Babić, B. Guberina, R. Horvat, H. Štefančić, *Phys. Rev.* **D65** (2002) 085002; A. Babić, B. Guberina, R. Horvat, H. Štefančić, *Phys. Rev.* **D71** (2005) 124041.
- [15] E. Bentivegna, A. Bonanno, M. Reuter, *JCAP* **01** (2004) 001.
- [16] B. Guberina, [arXiv:0707.3778](#) [gr-qc].
- [17] P.J. Steinhardt, in: Proc. of the 25th Anniversary Conference on Critical Problems in Physics, ed. V.L. Fitch, D.R. Marlow, M.A.E. Dementi (Princeton Univ. Pr., Princeton, 1997).
- [18] J. Grande, J. Solà and H. Štefančić, *JCAP* **0608** (2006) 011, [gr-qc/0604057](#).
- [19] J. Grande, J. Solà and H. Štefančić, *Phys. Lett.* **B645** (2007) 236, [gr-qc/0609083](#).
- [20] J. Grande, J. Solà and H. Štefančić, *J. Phys.* **A40** (2007) 6787, [gr-qc/0701090](#) 011; *J. Phys.* **A40** (2007) 6935, [astro-ph/0701201](#).
- [21] R.D. Peccei, J. Solà, C. Wetterich, *Phys. Lett.* **B195** (1987) 183; C. Wetterich, *Nucl. Phys.* **B302** (1988) 668; J. Solà, *Phys. Lett.* **B228** (1989) 317; *Int. J. of Mod. Phys.* **A5** (1990) 4225.
- [22] S. Cole et al, *Mon. Not. Roy. Astron. Soc.* **362** (2005) 505-534, [astro-ph/0501174](#).
- [23] O. Lahav et al., *Mon. Not. Roy. Astron. Soc.* **333** (2002) 961; L. Verde et al., *Mon. Not. Roy. Astron. Soc.* **335** (2002) 432.
- [24] R. S. Somerville, G. Lemson, Y. Sigad, A. Dekel, J. Colberg, G. Kauffmann, S. D. M. White, *Mon. Not. Roy. Astron. Soc.* **320** (2001) 289.
- [25] J. Fabris, I.L. Shapiro, J. Solà, *JCAP* **02** (2007) 016, [gr-qc/0609017](#).
- [26] L. Amendola, *Phys. Rev.* **D62** (2000) 043511; W. Zimdahl, D. Pavon, *Phys. Lett.* **B521** (2001) 133.

- [27] P. G. Ferreira, M. Joyce, *Phys. Rev.* **D58** (1998) 023503.
- [28] C.P. Ma , R.R. Caldwell, P. Bode , L.M. Wang, *Astrophys. J.* **521** (1999) L1.
- [29] E. V. Linder, A. Jenkins, *Mon. Not. Roy. Astron. Soc.* **346** (2003) 573.
- [30] R. Opher, A. Pelinson, astro-ph/0703779.
- [31] R. Opher, A. Pelinson, *Phys. Rev.* **D70** (2004) 063529.
- [32] P.J.E. Peebles, *Physical Principles of Cosmology* (Princeton University Press, 1993);  
T. Padmanabhan, *Structure formation in the Universe* (Cambridge University Press, 1993);  
A. R. Liddle, D.H. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge Univ. Press, 2000).
- [33] R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, *Phys. Rev. Lett.* **91** (2003) 071301;  
A. Melchiorri, L. Mersini, C.J. Odman, M. Trodden, *Phys. Rev.* **D68** (2003) 043509;  
H. Štefančić, *Phys. Lett.* **B586** (2004) 5; *Eur. Phys. J* **C36** (2004) 523; S. Nojiri, S.D.  
Odintsov, *Phys. Rev.* **D70** (2004) 103522; B. Feng, X. L. Wang, X. M. Zhang, *Phys.*  
*Lett.* **B607** (2005) 35.
- [34] B. McInnes, *JHEP* 0208 (2002) 029.
- [35] <http://snap.lbl.gov/>; <http://www.darkenergysurvey.org/>
- [36] P. Astier, *et al.*, *Astron. Astrophys.* **447** (2006) 31.